

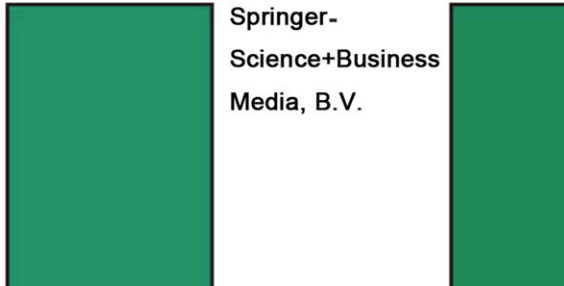
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Knowledge and Demonstration

Aristotle's *Posterior
Analytics*

by
Orna Harari



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KNOWLEDGE AND DEMONSTRATION

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Aristotle's Posterior Analytics

By

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*To the memory of Edna Harari
(1963-1985)*

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Introduction

This study explores the theoretical relationship between Aristotle's theory of syllogism and his conception of demonstrative knowledge. More specifically, I consider *why* Aristotle's theory of demonstration presupposes his theory of syllogism. In reconsidering the relationship between Aristotle's two *Analytics*, I modify this widely discussed question. The problem of the relationship between Aristotle's logic and his theory of proof is commonly approached from the standpoint of *whether* the theory of demonstration presupposes the theory of syllogism. By contrast, I assume the theoretical relationship between these two theories from the start. This assumption is based on much explicit textual evidence indicating that Aristotle considers the theory of demonstration a branch of the theory of syllogism. I see no textual reasons for doubting the theoretical relationship between Aristotle's two *Analytics* so I attempt to uncover here the common theoretical assumptions that relate the syllogistic form of reasoning to the cognitive state (i.e., knowledge), which is attained through syllogistic inferences. This modification of the traditional approach reflects the wider objective of this essay. Unlike the traditional interpretation, which views the *Posterior Analytics* in light of scientific practice, this study aims to lay the foundation for a comprehensive interpretation of the *Posterior Analytics*, considering this work from a metaphysical perspective. One of my major assertions is that Aristotle's conception of substance is essential for a grasp of his theory of demonstration in general, and of the role of syllogistic logic in particular.

Traditionally, the *Posterior Analytics* is regarded as the first elaborate theory of the structure of science. Aristotle's discussions of *epistêmê* therefore differ in their subject matter from his predecessors'. Plato, for instance, was occupied with the notion of knowledge but Aristotle is considered to have introduced a new subject for philosophical analysis: no less than science itself. As a result, Aristotle's theory of demonstration is usually detached from its historical context. The philosophical problems that he raises are assessed in light of modern philosophy of science rather than in light of pre-Socratic and Platonic conceptions of knowledge.

This approach is reflected in modern discussions on the meaning that the term *epistêmê* carries throughout the *Posterior Analytics*. In the past few decades various commentators have questioned the traditional rendition of *epistêmê* as "knowledge" or "science", arguing that in fact Aristotle's notion

of *epistêmê* is closer to the modern conception of understanding than to the modern conception of knowledge. This argument is based on a distinction between two facets of cognition: the grasp of certain content, associated with understanding and explanation, and the judgment as to the truth of this content, associated with knowledge and justification. The contention that the judgmental facet has little or no place in Aristotle's theory of demonstration leads to the identification of *epistêmê* with understanding.¹ Such an interpretation of one of the central terms in the *Posterior Analytics* stems from an evaluation of Aristotle's theory of knowledge, which takes the modern conception of knowledge as its paradigm. This study, by contrast, attempts to place Aristotle's conception of knowledge within its historical context, claiming that it does not presuppose the modern distinction between judgment and understanding. In accounting for Aristotle's conception of knowledge, I propose a distinction between two types of understanding – perceptual understanding and conceptual understanding – and I show that Aristotle's notion of knowledge presupposes this distinction.

Furthermore, the assumption that the *Posterior Analytics* deals with the structure of science has led Aristotle's commentators to interpret it from the perspective of scientific practice. A conspicuous expression of this approach is found in the controversy over the general objective of the *Posterior Analytics*. Presupposing that the *Posterior Analytics* deals with scientific methodology, Aristotle's modern commentators are puzzled by the discrepancy between his theory and scientific practice. It has been suggested that the *Posterior Analytics* is not a treatise on the methodology of science but on the organization and presentation of an achieved body of knowledge.² Other commentators cling to the traditional view, claiming that Aristotle's biological treatises do conform to the structure presented in the *Posterior Analytics*.³ Here I deal mainly with the discrepancy between Aristotle's theory and Greek mathematics. Through a comparative analysis of Euclid's *Elements* and Aristotle's theory of demonstration, I show that Aristotle imposes his conception of substance on mathematical notions, rather than forming his theory of demonstration according to mathematical practice. My analysis, which traces the reasons for this discrepancy in Aristotle's metaphysics, yields two conclusions. First it undermines the assumption that Aristotle's theory of knowledge aims to provide an adequate analysis of scientific practice; second, it shows that Aristotle's treatment of geometrical proofs is consistent with his metaphysical standpoint.

¹ See, for instance: Burnyeat, "Aristotle on Understanding Knowledge" (1981), p. 115.

² For example Burnes, "Aristotle's Theory of Demonstration" (1969), pp. 123-152.

³ A number of articles that express this view are collected in: Gotthelf and Lennox (eds.), *Philosophical Issues in Aristotle's Biology* (1987).

Apart from raising the problem of this discrepancy, the scientific approach to the *Posterior Analytics* fails to explain the role of syllogistic reasoning in the theory of demonstration. The rare occurrences of syllogistic inferences in Aristotle's scientific works and his contemporaries' scientific works have led to two different views of the role of syllogistic reasoning in the theory of demonstration. One view considers the theory of syllogism an incidental facet of the theory of demonstration. This interpretation rests on a developmental hypothesis, according to which the theory of demonstration was developed before the theory of syllogism.⁴ The other view is that the *Posterior Analytics* should not be regarded as *requiring* that science be formal but as offering a formal description of science. Still, the proponents of this view argue that although science should not actually be presented syllogistically, Aristotle's theory requires that its results can be put in syllogistic form.⁵ Despite the attempt to explain the role of syllogistic inferences in Aristotle's theory of demonstration, the latter view considers the syllogistic form indifferent to scientific results. In this interpretation, scientific results can be cast into syllogistic form without any modifications. This interpretation fails to say why Aristotle imposes such a narrow logical form on science, and it is also based on the modern assumption that logical forms are indifferent to contents. My examination of various attempts to reformulate Euclidean proofs syllogistically indicates that such formalizations *do* affect the content of mathematical proofs. More specifically, I show that syllogistic reformulations cannot assimilate construction steps, which constitute the core of a Euclidean proof.

My analysis of Aristotle's theory of syllogism provides an explanation for the difficulties involved in formulating geometrical proofs syllogistically by pointing to a fundamental difference between Aristotle's logic and modern logic. While the modern notion of logical entailment is based on truth-values, Aristotle's notion of entailment is shown here to be based on the relative extensions of the syllogistic terms. Generally, I argue that Aristotle's logic differs from modern logic in its objective. Aristotle's logic is based on the relation between the universal and the particular, and thus provides a general theory of the derivation of content from another. Modern logic formalizes the derivation of truth-value from another. Unlike modern logic, Aristotle's logic is not indifferent to the content of the inference. This study attempts to show that it is precisely this characteristic that makes syllogistic reasoning essential for Aristotle's theory of demonstration. In analyzing the demonstrative procedure, I argue that in demonstration the predicate of the conclusion undergoes a modification. This modification

⁴ See, e.g., Barnes, "Proof and Syllogism" (1981), pp. 17-59.

⁵ Gotthelf, "First Principles in Aristotle's *Parts of Animals*" (1987), pp. 167-198.

turns knowledge of the fact (which according to my analysis is a perceptual understanding) into knowledge of the reason why (which is a conceptual understanding). Syllogistic inference, which requires the same term to serve as the subject in one premiss and as the predicate in another premiss, is shown here to be the form of inference that makes such a modification possible.

The problem of the relationship between Aristotle's theory of syllogism and his theory of demonstration does not arise solely from the discrepancy between his theory and scientific practice. Aristotle's explicit contention that demonstration is a type of syllogism leads one to expect that the theory of syllogism imposes constraints on the formal structure of the first principles of demonstration. However, *prima facie*, Aristotle's discussion of the first principles of demonstration in the *Posterior Analytics* does not seem to accord with the syllogistic requirements. In the *Posterior Analytics* I.2 Aristotle classifies the first principles of demonstration under three groups: common axioms, hypotheses, and definitions. Among the common axioms, Aristotle includes the law of the excluded middle and the proposition that if equals are subtracted from equals, the remainder is equal. These propositions, which are classified as demonstrative principles, manifestly do not accord with the four forms of syllogistic premisses. That is, they are not predicative propositions of the forms: "A belongs to all B", "A belongs to some B", "A belongs to no B" and "A does not belong to some B". A similar problem arises with regard to hypotheses; the *Posterior Analytics* provides evidence that hypotheses are existential propositions of the form "A exists" rather than predicative propositions. Definitions, conversely, accommodate the predicative form; however, Aristotle's view of the role of definitions in demonstration is unclear. On the one hand, he includes definitions among the *principles* of demonstration, while on the other he states explicitly that definitions are not included among the *premisses* of demonstration (76b35-37). The first two chapters of this essay address this problem, and set out to accommodate Aristotle's account of first principles with his theory of syllogism.

To pave the way to an interpretation of Aristotle's account of first principles, this study opens with an analysis of the various occurrences of the term *archê* (i.e., first principle) throughout the *Posterior Analytics*. This analysis leads to the conclusion that in the *Posterior Analytics* the term *archê* has two meanings: one refers to the intellect (*nous*), and the other to the immediate propositions that serve as the premisses of demonstration. Furthermore, by comparing Aristotle's discussion of first principles in the *Posterior Analytics* I.2 with his discussions in the *Posterior Analytics* I.7,10, I argue that Aristotle's distinction between hypotheses and definitions is not identical with that between primitive and derived terms, which is made in the

Posterior Analytics I.7,10. According to my textual analysis, definitions and hypotheses are included among the first principles of demonstration, whereas in the *Posterior Analytics* I.10 Aristotle regards only the primitive terms as first principles. Hence, I argue that the derived terms are not regarded as first principles but merely as assumptions, required for constituting a demonstrative branch of knowledge. In light of these distinctions I discuss separately the intellect as the cognitive faculty that grasps primitive terms and the immediate propositions that serve as the premisses of demonstration.

My interpretation of Aristotle's account of first principles opens with a discussion of the intellect and the inductive process that leads to the first principles. Aristotle's discussion of induction in the *Posterior Analytics* II.19 differs from his other discussions of this procedure. In other contexts Aristotle considers induction a form of inference that establishes universal propositions; in the *Posterior Analytics* II.19 the inductive process leads to the cognition of universal concepts, such as "man" and "animal". Concerning whether Aristotle's account of the acquisition of first principles in the *Posterior Analytics* II.19 holds for concepts or propositions, I analyze his notion of induction. My survey of the various uses of the term *epagôgê* in the Aristotelian corpus results in a distinction between two senses of induction: an argumentative sense, discussed mainly in the *Topics*, and a cognitive sense, introduced in two passages, one from the *Prior Analytics* II.21 and the other from the *Posterior Analytics* I.1. According to my analysis of these passages, argumentative induction yields a conviction (*pistis*) as to the truth-value of a universal proposition, while cognitive induction leads to a qualitative modification of certain content; more specifically it leads from an acquaintance with matter to an acquaintance with forms.

In my interpretation of the *Posterior Analytics* II.19, I show that this discussion presupposes the cognitive notion of induction, which is presented in the *Prior Analytics* II.21 and the *Posterior Analytics* I.1. This interpretation implies that Aristotle's discussion of induction in the *Posterior Analytics* II.19, like his discussions in the *Prior Analytics* II.21 and the *Posterior Analytics* I.1, forms his response to Plato's theory of recollection. Consequently, his notion of induction can be understood as accounting for learning, rather than as establishing an empirical foundation of knowledge. Further, this interpretation leads to the conclusion that the inductive procedure, described in the *Posterior Analytics* II.19, leads to generic concepts and not to self-evident propositions. Hence, the intellect is not an intuitive apprehension of basic truths, as certain modern epistemologies suggest, but an understanding of basic contents, i.e., concepts. This contention reflects Aristotle's general approach to knowledge. Unlike

modern epistemology, Aristotle bases his theory of knowledge on an understanding of basic concepts, rather than on basic judgments. In the subsequent chapters I show that the priority of understanding over judgment determines Aristotle's views of the immediate premisses of demonstration, the logic of demonstration, the object of knowledge, and ultimately it determines his concept of knowledge itself.

Chapter two examines the formal structure of the immediate premisses of demonstration (i.e., hypotheses) and their modal status. The major problem arising from Aristotle's account of hypotheses is how their existential import accords with their propositional form. Seemingly, Aristotle's contention that hypotheses differ from definitions in stating that "something is or is not" indicates that hypotheses are existential propositions of the form "X exists". Yet this contention does not accord with the syllogistic structure of demonstrations, whose premisses are predicative propositions of the form "X is Y". In an attempt to accommodate Aristotle's view of hypotheses to syllogistic reasoning, I interpret his discussion of them in light of his discussion of being in *Metaphysics* V.7. I argue that hypotheses imply being in the sense of being true. So in my interpretation hypotheses and definitions are both predicative propositions, which differ in their truth-values; that is, hypotheses may be either true or false, while definitions cannot be false.

Further, Aristotle's account of hypotheses seems to be at odds with their modal status. On the one hand, their characterization as "one part of a contradiction" implies that hypotheses have an intelligible, albeit false alternative. On the other hand, Aristotle's contention in the *Posterior Analytics* I.4, 6 that the premisses of demonstration are necessary implies that the negations of hypotheses are impossible rather than simply false. I examine how hypotheses can have an intelligible alternative and yet be necessary propositions. By analyzing Aristotle's conception of the law of the excluded middle in *Metaphysics* IV and in the *Posterior Analytics* I.11, I show that from Aristotle's point of view, subordination to the principle of the excluded middle is a mark of essential predications. Hence, having a truth-value or, in Aristotle's terms, being "one part of a contradiction" is tantamount to being a necessary, i.e., "in-itself", proposition.

Next my analysis of the modal status of hypotheses uncovers the crucial role of definitions in Aristotle's theory of demonstration. Through an analysis of Aristotle's discussion of error in the *Posterior Analytics* I.5, I show that Aristotle distinguishes knowledge from error in terms of a distinction between a demonstration that establishes an essential relation between the premisses and the conclusion, and a demonstration that fails to establish such a relation. Accordingly, I argue that the role of definitions in Aristotle's theory of demonstration is not confined to specifying the meanings of the basic terms. Instead, since demonstrations are based on

essential relations between the premisses and the conclusion, definitions determine demonstrative entailment as well.

In chapter three I elaborate on the logical relations subsisting in syllogistic inferences. To explicate Aristotle's notion of logical entailment, I consider his reasons for excluding hypothetical deductions from his theory of syllogism. I argue that hypothetical deductions are excluded from the theory of syllogism since they are based on an arbitrary agreement as to the truth-values of the hypothetical premisses. Aristotle's treatment of hypothetical deductions reflects the basic difference between his logic and modern logic. Whereas the distinction between a proposition and its truth-value is fundamental for the modern notion of entailment, Aristotle's logic presupposes a dependence of the truth-value upon the propositional content.

Aristotle's conception of the relationship between a proposition and its truth-value affects his notion of logical entailment. In modern logic a conclusion is logically entailed by the premisses in all cases except when the premisses are true and the conclusion is false. An obvious consequence of this definition is that the truth-value of the conclusion cannot be determined, given that the premisses are false. By contrast, in the *Prior Analytics* II.2-4 Aristotle specifies the conditions under which a true conclusion is to be drawn from false premisses. The very possibility of determining the truth-value of a conclusion drawn from false premisses indicates that Aristotle's notion of logical entailment is different from the modern one. By examining Aristotle's detailed analysis of inferences from false premisses, I argue that apart from being based on a relation between truth-values, syllogistic entailment is based on a relation of subordination and coordination between the syllogistic terms. Consequently, I show that Aristotle's notion of entailment is narrower than the modern notion of entailment, in regarding as non-syllogistic certain inferences that modern logic considers valid.

My interpretation of Aristotle's theory of syllogism leads to the conclusion that his logic and his theory of demonstration are based on a common presupposition. In both theories Aristotle ascribes priority to the concept over the judgment. In formulating the theory of syllogism, Aristotle views as constitutive the relation between the universal and the particular, which is a generalization of the *conceptual* relations subsisting in predications. Likewise, in accounting for the first principles of knowledge, he views definitions as determining the demonstrative relation between the premisses and the conclusion. Hence, the priority of concepts or essences over judgments is a common presupposition of Aristotle's logic and his theory of proof.

In the fourth chapter I focus on the discrepancy between Aristotle's theory of demonstration and scientific practice and find the reasons for this discrepancy in his conception of the objects of knowledge. With respect to

Aristotle's discussion of mathematical objects in *Metaphysics* XIII. 1-3, I consider the reasons why Aristotle does not completely dismiss their substantiality even though he views them as attributes belonging to the category of quantity. I show that Aristotle's ambiguous attitude to mathematical objects is an expression of his attitude to attributes in general. Seeing Aristotle's discussion of mathematical objects in this light, I show that their characterization as "the results of abstraction" refers to the method by which quantitative attributes are defined. This characterization enables Aristotle to endow quantitative attributes with two of the characteristics of substances, i.e., separability and essence, thereby turning them into knowable objects.

In endorsing the method of abstraction for defining attributes, Aristotle modifies his claim, made in *Metaphysics* VII.4, that only substances can be defined (1030a11-20). I trace the reasons for this modification to Aristotle's attempt to accommodate the objects of knowledge to demonstrative syllogisms, which require essences as their starting point. In substantiating this contention, I analyze Aristotle syllogistic formulations of geometrical proofs and the geometrical examples presented in his discussion of demonstrative premisses in the *Posterior Analytics* I.4. In both cases Aristotle is consistent in adopting the approach of *Metaphysics* VII.4; that is, he attempts to accommodate the various relations that hold for attributes with the relations between genera and species that hold for substances. It follows from my analysis of syllogistic logic that this attempt is tantamount to an attempt to accommodate the objects of knowledge, which are attributes, to syllogistic reasoning which presupposes the relation between genera and species. Hence, my analyses of syllogistic logic and of Aristotle's view of the objects of knowledge disclose the fundamentality of syllogistic reasoning for Aristotle's theory of knowledge.

Chapter five discusses the relationship between Aristotle's concept of knowledge and the demonstrative procedure by which knowledge is attained. My starting point here is the distinction, made at the beginning of the second book of the *Posterior Analytics*, between the two stages of demonstration: (1) the preliminary stage that concerns the fact and the question "whether it is?" and (2) the final stage that deals with reasons and the question "what it is?" (89b36-90a1). Regarding the preliminary stage, I consider how definitions can determine being and argue that they do so by distinguishing a certain entity from other entities. Such a definition does not imply existence in the modern sense but indicates that an entity is a certain "this" (*tode ti*). I show that from Aristotle's point of view definitions imply being by pointing out that a certain entity is a substance.

As for the second demonstrative stage, I construe the distinction between knowledge of the fact and knowledge of the reason why as anchored in

Aristotle's distinction between things that are more familiar to us and things that are more familiar by nature. As a result, I show that the two demonstrative stages differ in yielding different understandings of the same phenomenon; knowledge of the fact yields a perceptual understanding, whereas knowledge of the reason why yields a conceptual understanding. Both types of understanding are attained through definitions that capture the universal facet of a given phenomenon; the perceptual understanding grasps the typical features of a phenomenon by appealing to perceptual consideration, while conceptual understanding defines a phenomenon by subsuming it under its genus and *differentiae*.

Having characterized the two stages of demonstration, I next consider the relationship between demonstration and essence. Here I claim that demonstration of the essence is not a demonstration that concludes a definition but one that reveals the essence indirectly. According to my interpretation, the demonstrative procedure aims to exhibit the essence of the major term (i.e., the predicate of the conclusion) by subsuming it under the middle term. In light of this interpretation I show that through demonstrations the major term undergoes a modification. Prior to the demonstration the major term bears a perceptual meaning, i.e., the meaning that is delineated in the answer to the question "whether it is". The subsumption of the major term under the middle term, through demonstration, yields an answer to the question "what it is"; it thereby endows the major term with the conceptual meaning, expressed in a definition by genus and *differentiae*. Consequently, I argue that Aristotle views knowledge as a conceptualization, attained through syllogistic inferences.

In conclusion, I show that Aristotle's theory of demonstration has been developed as a counterpart to his theory of substance. According to Aristotle's theory of substance knowledge of substances (i.e., grammatical and ontological subjects) is attained through definitions. The theory of demonstration specifies how a similar mode of cognition is applicable to the objects of knowledge, which grammatically and ontologically are attributes. This attitude determines Aristotle's theory of demonstration, which I here view as an attempt to accommodate scientific knowledge to the theory of substance.

In attempting an interpretation, I make various assumptions. First, from the viewpoint of methodological considerations I assume that the Aristotelian corpus expresses a coherent view rather than reflecting different stages in the development of Aristotle's philosophy. Unlike many interpreters, I do not consider developmental hypotheses to belong to the explanatory category. Aristotle's philosophy certainly underwent various modifications in different issues, and indicators of these revisions can clearly

be found in his writings. However, without evidence of absolute dating, chronological considerations cannot be taken into account. The main problem with a developmental approach is that any dating of Aristotle's writings rests on a prior assumption of the course of the development of Aristotle's philosophy. Since such a presupposition is based on interpretation, chronological considerations render the interpretation of Aristotle circular.

Furthermore, although such a methodological assumption neglects the historical development of Aristotle's philosophy, it does provide the means of establishing an interpretation based solely on textual evidence. With the assumption that Aristotle's writings form a coherent whole, an interpretation can be based on as much textual evidence as can be found in the Aristotelian corpus. The developmental hypothesis, by contrast, reduces the scope of textual evidence to treatises assumed to be written before or at the same time as the treatise under discussion. An interpretation that emerges from the text being more reliable, the non-developmental hypothesis facilitates a better understanding of Aristotle's philosophy at this distance in time.

Second, although one cannot ignore the possibility of textual corruption, I assume that a coherent interpretation can be given on the basis of the edited text. Likewise, I assume that Aristotle's philosophy is coherent and justifiable so I assume that the text is free from textual or conceptual errors. Although this assumption may not reflect reality, it is indispensable for establishing an adequate interpretation. Errors can be detected only in light of a prior assumption as to the correct view. Hence, claims of textual or conceptual errors virtually reflect the discrepancy between the interpreter's prior assumptions and the interpreted data. An interpretation that assumes error as an explanatory category reveals the interpreter's evaluation or disapproval rather than his or her understanding. In regarding conceptual discrepancies as stemming from unsatisfactory prior assumptions, I attempt to reveal those prior assumptions that render the text coherent.

Third, I assume that philosophical ideas cannot be detached from their historical background, since like any other cultural manifestation they undergo changes throughout history. This assumption determines the final aim of my interpretation, which so to speak is an inquiry into presuppositions. By "presupposition" I mean prior assumptions that are taken for granted, and are therefore considered unquestionable. Adopting an historical approach, I assume that Aristotle's philosophy and modern philosophy do not share the same presuppositions. My interpretation is aimed at uncovering Aristotle's presuppositions and comparing them with modern presuppositions. In so doing, I follow two major rules. (1) I do not employ modern devices such as the notation of modern symbolic logic and modern algebra, in interpreting Aristotle's logic and Greek mathematics; and

(2) I base my interpretation solely on textual evidence since I regard external philosophical considerations as stemming from modern presuppositions, hence less reliable.

Chapter One

Intellect as a First Principle

1.1 THE AMBIGUITY OF THE TERM ARCHÊ

The *Posterior Analytics* opens with the assertion that all teaching and learning stem from pre-existing acquaintance (*proïparchousês gnôseôs*) (71a1-2), which serves, according to Aristotle, as the first principle (*archê*) of demonstration and knowledge. Although this assumption is crucial for Aristotle's theory of demonstrative knowledge, the details of the nature and the role of the first principles remain obscure. The first problem that arises from Aristotle's discussion of the first principles concerns what the principles of knowledge are. In several passages throughout the *Posterior Analytics* Aristotle introduces a triple distinction of the principles found in demonstrative knowledge (I.1, 71a11-16; I.2, 72a14-24; I.7, 75a39-b2; I.10, 76a31-b10; 76b11-16). In the *Posterior Analytics* I.1, I.7, and I.10 Aristotle draws a distinction between common principles, which are presupposed by any branch of knowledge, and proper principles, which are divided into those that state what a thing is and those that state that a thing is. In the *Posterior Analytics* I.2 Aristotle introduces the technical terms axioms, definitions, and hypotheses in distinguishing, respectively, principles that ought to be assumed by any learner, principles that state what a thing is, and principles that state that a thing is. At first glance, the triple distinction made in the *Posterior Analytics* I.1, I.7, and I.10 seems to coincide with the distinction between axioms, definitions, and hypotheses, made in the *Posterior Analytics* I.2. In the *Posterior Analytics* I.10, for instance, Aristotle says:

For every demonstrative knowledge has to do with three things: what it posits to be (these form the genus of what it considers the attributes that belong to it in itself); and what are called the common axioms, the primitives from which it demonstrates;

and thirdly the attributes, of which it assumes what it signifies (76b11-16).¹

From the assumption that in this passage Aristotle implicitly refers to the triple distinction of axioms, definitions, and hypotheses, it seems plausible to identify the assumption of the being of the genus with hypotheses and the assumption of the significance of the attributes with definitions. But on closer scrutiny the relevant passages do not endorse this parallelism unless one accuses Aristotle of a double confusion: between premisses and conclusions and between terms and propositions.

In the *Posterior Analytics* I.2 the triple distinction of axioms, definitions, and hypotheses applies to the premisses of demonstration, whereas in the *Posterior Analytics* I.7 and I.10 the distinction that seems equivalent to that between definitions and hypotheses in fact applies to the subject and the predicate of the conclusion. In the *Posterior Analytics* I.7, as well as in I.10, the distinction between common principles and proper principles coincides with a distinction between *what* knowledge is about and *from what* knowledge stems (*ex hōn*) (75a42). In the *Posterior Analytics* I.2, by contrast, axioms, definitions, and hypotheses are all classified as syllogistic premisses (72a14-15); that is, they fall conjointly under the category of “from what”. Hence, endorsement of the parallelism between the *Posterior Analytics* I.2 and the *Posterior Analytics* I.10 leads to an inconsistent account of the first principles. Under the supposition that the *Posterior Analytics* I.2, I.7, and I.10 are parallel, the term *archê* has to be understood to refer to the premisses of demonstration as well as to the components of the demonstrative conclusion.

This problem of the scope of application of the term *archê* stems from the assumption that the distinction between hypotheses and definitions is identical to the separate distinction made in the *Posterior Analytics* I.10 between primitive (*ta prôta*) and derived (*ta ek toutôn*) terms. Such identification, however, does not accord with Aristotle’s characterization of these terms. Unlike the distinction between hypotheses and definitions, the distinction between primitive and derived terms is *not* a distinction between two types of first principles. In the opening paragraph of the *Posterior Analytics* I.10 Aristotle says:

I call principles in each genus those which it is not possible to prove to be. Now both what the primitives and what the things

¹ In quoting Aristotle, I have generally chosen to follow the translations offered in Barnes (ed.), *The Complete Works of Aristotle* (1984). In certain cases I have revised the translation for the sake of precision or clarity.

dependent on them signify is assumed; but that they are must be assumed for the principles and proved for the rest. (76a31-34)

According to this passage, a principle is a term whose being is indemonstrable. This very characteristic distinguishes the primitive terms from the derived terms, whose being has to be proved. It follows that the derived terms are not principles, so this passage undermines the parallelism between hypotheses and definitions on the one hand, and primitive and derived terms on the other hand. Hypotheses and definitions are alike regarded as principles (*archai sullogistikai*) while the derived terms are not principles at all.

To reject the commonly held identification of the distinction between hypotheses and definitions with that between primitive and derived terms has the advantage of clarifying the structure of the *Posterior Analytics* I.10 as a whole.² More particularly, it helps to clarify the ambiguity about whether Aristotle views the first principles as propositions or terms. In this chapter Aristotle refers to the principles of knowledge in three successive paragraphs; in the first he discusses the “principles in each genus”, distinguishing primitive and derived terms. Among the primitives, whose being and signification has to be assumed, Aristotle includes terms such as “unit” and “magnitude”, and among the derived terms, whose signification has to be assumed and their being has to be proved, he includes terms such as “straight” and “triangle” (76a31-36). The third paragraph of the *Posterior Analytics* I.10 similarly deals with terms and not with propositions. In this paragraph Aristotle refers to terms like “unit” and like “even” and “odd” in drawing the grammatical distinction between predicates that belong essentially to a subject and the subject to which these predicates belong (76b3-10). The distinction made in this paragraph, between stating that something is and stating what something is suggests that the principles discussed here are the “principles in each genus” discussed in the first paragraph, i.e., the subject and the predicate of the conclusion.

In the second paragraph Aristotle discusses “things *used* in demonstrative knowledge”. Here he distinguishes common and proper assumptions.³ The proposition “line is such and such” exemplifies proper assumptions and the

² The identification of the distinction between hypotheses and definitions with the distinction between primitive and derived terms is presupposed in all interpretations known to me. It can be found in: Barnes, *Aristotle's Posterior Analytics* (1993), pp. 100, 137; von Fritz, “Die *Archai* in der griechischen Mathematik” (1971), p. 348; Hintikka, “On the Ingredients of an Aristotelian Science” (1972), pp. 67-69; Landor, “Definitions and Hypotheses in *Posterior Analytics* 72a19-25 and 76b35-77a4” (1981), pp. 308-318; McKirahan, *Principles and Proofs -- Aristotle's Theory of Demonstrative Science* (1992), p. 44; Ross, *Aristotle's Prior and Posterior Analytics* (1949), p. 55 nn. 6, 7.

³ Aristotle does not use the term *archê* in this paragraph.

proposition “if equals are taken from equals, the remainders are equals” exemplifies common assumptions (76a37-b2). In this passage, in contrast to the preceding one, Aristotle refers to predications and not to terms. Therefore the distinction made in the first paragraph, which is exemplified by the assertion “unit is” cannot be equivalent to the predicative proposition “line is such and such”. So the distinction introduced in the second paragraph (i.e., “the things used in demonstration”) seems not to coincide with the distinction between primitives and derived terms, which is introduced in the first and third paragraphs.⁴

Thus, throughout the first section of the *Posterior Analytics* I.10, Aristotle deals with two topics: the premisses of demonstration that are used in demonstrative proofs and the genus and its attributes, about which knowledge is. Such an interpretation of *Posterior Analytics* I.10 has the merit of averting the conclusion that Aristotle views both the premisses of demonstration and the constituents of the demonstrative conclusion as first principles. Further, it does not treat phrasing differences, like “unit is” and “line is such and such”, as unimportant subtleties.

To avoid inconsistencies I regard as first principles only ingredients that are denoted explicitly by the term *archê*. This leaves us with two types of first principles: the primitive terms denoting the genera, whose being is indemonstrable and the principles of demonstrative syllogism, i.e., the axioms, the definitions, and the hypotheses, which are discussed mainly in the *Posterior Analytics* I.2. By contrast, the derived terms are the predicate of the demonstrative conclusion, whose meaning has to be assumed and whose being has to be proved. These derived terms are discussed in the *Posterior Analytics* I.1 (71a14-17), I.10 (76a31-36; 76b3-16) and in the first ten chapters of the second book of the *Posterior Analytics*. I offer a detailed analysis of the derived terms in chapter 5.

1.2 TWO SENSES OF ARCHÊ

The major problem of interpreting Aristotle’s notion of first principles concerns their very nature. Aristotle views knowledge as deductive: knowledge is attained through demonstration (*apodeixis*), which is an inference having a syllogistic form. Since syllogisms are inferences from categorical propositions, these not surprisingly are found among Aristotle’s examples of the first principles. The examples that Aristotle gives for axioms

⁴ In 76b6 Aristotle claims that with regard to the proper items one ought to assume “that they are” and “that they are such and such”. This claim is parallel to the requirement that with regard to the primitives, both being and signification should be assumed.

and definitions are, respectively, the law of the excluded middle (71a13-14) and the proposition: “unit is what is quantitatively indivisible” (72a23-24). Even though commentators differ as to whether hypotheses are existence claims or predicative propositions, the characterization of a hypothesis as one part of a contradiction indicates that hypotheses are propositions as well.⁵

In the last chapter of the *Posterior Analytics* Aristotle considers how the first principles become known. In the light of his previous discussions one would expect him to account for the attainment of knowledge of primitive propositions. However, his discussion in the *Posterior Analytics* II.19 concerns only the cognition of universal *concepts* such as “man” and “animal”. This discrepancy between the conceptual and the propositional accounts of the first principles finds expression in passages outside the *Analytics* as well. Listing the various meanings of the term *archê* in *Metaphysics* V.1, Aristotle claims that it is a hypothesis from which demonstration stems (1013a14-16). Yet many other passages suggest that only definitions are the principles of demonstration.⁶ I argue that this duality is an expression of a distinction between two different senses of the term *archê*, which corresponds to the distinction between concepts and propositions. Textual evidence in support of this view follows.

In the *Posterior Analytics* I.24 Aristotle argues that a universal demonstration is better than a particular one. He claims that a universal proof is adduced through a middle term that is *nearer* to the principle (*egguterô tês archês*). He states that the phrase “nearer to the principle” means: “the immediate is nearer to the principle and indeed it is a principle” (86a13-15). The phrase “nearer to the principle” implies a distinction between two senses of the term *archê*, for otherwise the principle would be nearer to itself. Aristotle’s argument for the priority of a universal demonstration seems to be based on a distinction between two types of first principles: one refers to an ultimate principle and the other to “the immediate”, which is nearer to the ultimate principle.

This conclusion is in fact confirmed in two other passages, in which two senses of the term *archê* are distinguished. The first passage appears in the *Posterior Analytics* I.23:

And just as in other cases the principle is simple, though it is not the same everywhere – but in weight it is the ounce, in song the semitone, and in other cases other things – so in syllogism

⁵ The expression “one part of a contradiction” (*thateron morion tês antiphraseôs*) is a characterization of syllogistic premisses. See, for example, *Prior Analytics* I.1 24a23.

⁶ *Topics*, 158a33,39; *Metaphysics*, 998b5, 29, 1034a30-32, 1078b23; *Nicomachean Ethics*, 1142a26, 1143a26,b2.

the one is the immediate premiss and in demonstration and knowledge it is the intellect. (84b37-85a1)⁷

Aristotle, then, distinguishes the principles of knowledge from the principles of syllogism; the principle of knowledge is the intellect (*nous*) and the principle of the syllogistic inference is an immediate proposition.

The same distinction is made in the *Posterior Analytics* I.33, where Aristotle says that contingency, namely the possibility of being otherwise belongs

...neither (*oude*) to the intellect. By intellect I mean a principle of knowledge. Nor yet (*oude*) to indemonstrable knowledge, this is a belief in an immediate proposition. (88b35-36)⁸

Construing the Greek particle *oude* as “*id est* not”, Ross and Barnes have argued that indemonstrable knowledge and intellect are identical.⁹ Such an interpretation might be plausible on the basis of Ross’s reading, which conceals the distinction by altering the punctuation. In Ross’s edition “intellect” (*nous*) does not appear as distinct from “indemonstrable knowledge” (*epistêmê anapodeiktos*): the latter expression (i.e., indemonstrable knowledge) appears as glossing the former. Bekker’s punctuation, by contrast, uses periods to separate the phrase *oude nous* from the phrase *oud’ epistêmê anapodeiktos*. In Bekker’s edition, intellect and indemonstrable knowledge appear as two distinct things. Bekker’s edition, unlike Ross’s, accords better with the common meaning of the construction *oude...oude*, which marks strong opposition. So “intellect” and “belief (*hupolêpsis*) in an immediate proposition” seem to refer, in this passage, to two different things.

In light of this evidence, I offer separate accounts for Aristotle’s view of the intellect and immediate premisses. These discussions reveal both the theoretical rationale behind the distinction between the intellect and the immediate premiss and the epistemological presuppositions that underlie Aristotle’s theory of knowledge.

⁷ Barnes’s translation is revised. Barnes omits, with no textual support, the words “*protasis amesos*”. His alternative translation “so in deduction it is the unit” makes no sense. This omission probably stems from reluctance to admit the distinction between the two senses of *archê*. [On this see Barnes, *Aristotle’s Posterior Analytics* (1973), p. 199.]

⁸ My translation here is based on Bekker’s edition.

⁹ Ross, *Aristotle’s Prior and Posterior Analytics* (1949), pp. 606-607; Barnes, *Aristotle’s Posterior Analytics* (1973), p. 199.

1.3 INDUCTION AND INTELLECT

At the beginning of the *Posterior Analytics* II.19 Aristotle poses two questions: (1) How do we become familiar with the first principles? (2) By means of what mental state (*hexis*) does one get to know them (99b18)? The first question is answered by a description of a process leading to the first principles, through sense perception, memory, and experience. This description ends, “thus it is clear that is necessary for us to become familiar with the primitives by induction” (*epagôgê*) (100b4). In answering the second question, Aristotle identifies the mental state that grasps the first principle with the intellect (*nous*). According to Aristotle, “induction” is the way by which the first principles are grasped and the intellect (*nous*) is considered the mental state of grasping these principles. The distinction between these two questions implies that induction and intellect are not both a means of acquiring the first principles, but that they relate to each other as do a procedure and its respective cognitive state.¹⁰ The relation between intellect and induction is analogous to the relation between knowledge and demonstration; knowledge and intellect are cognitive states, while demonstration and induction are the procedures leading to these cognitive states.¹¹ This correlation between induction and intellect has a bearing on the question of whether the intellect grasps propositions or concepts. In the following, I elaborate on this.

Aristotle’s discussions of induction outside the *Posterior Analytics* II.19 seem to support the propositional interpretation of the first principles. The description of the advance from the particular to the universal, which is given in the *Posterior Analytics* II.19, corresponds with Aristotle’s discussion in *Metaphysics* I.1, where he explains how a human being is capable of forming a universal judgment (*hupolêpsis*). In both passages Aristotle portrays a similar process, which begins with perception, passes through memory and results in experience (100a3-9; 980b25-981a7).¹² Yet the identity between these two processes does not hold for their results. The process described in *Metaphysics* I.1 leads to universal judgments, such as “this medicine has done good to all persons”, whereas in the *Posterior*

¹⁰ Cf. Barnes, *Aristotle’s Posterior Analytics* (1973), pp. 260, 270.

¹¹ Cf. Kahn “The Role of *Nous* in the Cognition of First Principles in *Posterior Analytics* II.19” (1981), pp. 385-414.

¹² In what follows I show that Aristotle’s discussion in *Metaphysics* I.1 has no bearing on induction.

Analytics II.19 the process leads to universal concepts such as “man” or “animal”.¹³

Further, induction is held by Aristotle to be the way that leads to the immediate premisses of demonstration. For instance, in the *Nicomachean Ethics* VI.3 Aristotle says that the principles from which the syllogism proceeds are acquired by induction (1139b28-31). Although the *Nicomachean Ethics* does not state explicitly that the first principles are propositions, this passage supports the propositional interpretation, since it states that the premisses of syllogisms (which are propositions) are attained by induction. Moreover, Aristotle’s most elaborate discussion of induction in the *Prior Analytics* II.23 suggests that induction is a means of establishing primitive and immediate propositions. In the *Prior Analytics* II.23 induction is characterized as an inference, in which a relation between the major term and the middle term is deduced by means of the minor term. Such a deduction can be made only when the minor term and the middle term are convertible, so that the minor term ought to be understood as consisting of all the particular instances (68b27-28). Since this induction, according to Aristotle, is a proof of primary and immediate premisses, an enumeration of all the cases seems indispensable for establishing a demonstration on solid ground. It might seem that the so-called “perfect induction”, which establishes an immediate premiss, is the induction leading to the first principles, discussed in the *Posterior Analytics* II.19.

However, the identification of the induction discussed in the *Posterior Analytics* II.19 with the syllogistic induction of the *Prior Analytics* II.23 does not accord with the above correlation between induction and intellect. In the *Topics* I.12 induction is defined as an argument (*logos*) that advances from the particular to the universal (105a10-14). Further, Aristotle calls the inductive argument, which is discussed in the *Prior Analytics* II.23, “a syllogism from induction” (68b15). From these discussions, induction like syllogism is a certain inference or argument. Yet this characterization of induction is at odds with the identification of the intellect with the state that grasps the first principles. In the *Posterior Analytics* II.19 Aristotle characterizes knowledge as accompanied by inference (*meta logou*), in arguing that there is no knowledge of the first principles (100b10-11). Although in this passage Aristotle does not explicitly claim that the cognition of the first principles by the intellect is not argumentative, the distinction between knowledge and intellect implies that this is one of the characteristics that distinguish these two mental states. This conclusion finds

¹³ On the basis of such a comparison Ross concludes that the formation of general concepts and the act of grasping universal propositions are alike only in being inductive [Ross, *Aristotle’s Prior and Posterior Analytics* (1949), p. 675].

support in the *Nicomachean Ethics* VI.11, where Aristotle says that the ultimate and primitive terms are grasped through the intellect and not by means of argument (*nous esti kai ou logos*) (1142a36-1143b1). The assumption that induction in the *Posterior Analytics* II.19 refers to the inductive inference that establishes immediate premisses renders incompatible Aristotle's explicit contentions that the first principles are grasped by induction and that the intellect is the mental state that grasps them. From this assumption, the intellect as a non-argumentative state operates through induction, which is a type of argument. Aristotle's modern commentators attempt to resolve this discrepancy by explaining away either the role of induction or the role of the intellect in grasping the first principles.¹⁴ In contrast to these views, I attempt to maintain both the role of the intellect and the role of induction in the *Posterior Analytics* II.19 by showing that the term "induction" is employed there in a non-argumentative sense.

I.4 INDUCTION

In his commentary on Aristotle's *Prior and Posterior Analytics*, Ross criticizes Trendelenburg's view, arguing that Aristotle's notion of induction is indeed related to the modern use of the term, in being a form of inference and not merely an act of adducing instances. Ross claims that the root idea of Aristotle's notion of induction is "an advance from one or more particular judgments to a general one".¹⁵ Although this feature characterizes induction in general, Ross maintains that Aristotle has no single unifying conception of induction. Analyzing the different occurrences of the word *epagôgê* and its inflections throughout the Aristotelian corpus, Ross distinguishes the following three technical uses of this term: (1) dialectical induction, which is a type of inference based on particular instances, that yields merely a belief in a general principle; (2) perfect induction, which is a *valid* argument from the particular to the universal; (3) intuitive induction, which is an insight, by

¹⁴ Hamlyn "Aristotelian *Epagoge*" (1976), p. 181 minimizes the role of induction in Aristotle's account of the acquisition of first principles. A similar view is held by Kall, *On Intuition and Discursive Reasoning in Aristotle* (1988), pp. 27-31; 44-53. Barnes takes the second position, claiming that *nous* has no bearing on Aristotle's account of the means by which the first principles are acquired [Barnes, *Aristotle's Posterior Analytics* (1973), p. 268].

¹⁵ Ross, *Aristotle's Prior and Posterior Analytics* (1949), p. 48 (my emphasis).

which one passes from knowledge of particular instances to knowledge of a general principle.¹⁶

When induction is conceived of as a form of argument, the expression “intuitive induction” appears perplexing. Induction as an argument is a logical procedure i.e., a process that involves mediation, methods, and rules, but on the other hand induction, being intuitive, is held to be an insight or an immediate apprehension, which is essentially a non-discursive act. This ambiguity is another expression of the discrepancy between intellect and induction. To use Barnes’s metaphor, the *Posterior Analytics* II.19 is Janus-faced; it tends towards empiricism and at the same time towards rationalism. In my view, however, the tension between intellect and induction is not intrinsic to Aristotle’s notion of induction but stems from interpreters’ misconception of the notion of induction employed in the *Posterior Analytics* II.19. To resolve this discrepancy I suggest a different classification of induction from Ross’s. Within the general characterization of induction as an advance from the particular to the universal, Aristotle, in my interpretation, uses induction in two senses: one is argumentative and the other is cognitive.¹⁷ By addressing first Ross’s distinction between dialectical induction and perfect induction, I show that these two types of induction are not categorically different.

Aristotle frequently says that conviction is gained either by syllogism or by induction.¹⁸ This characterization of induction as a means of conviction constitutes the core of the argumentative sense of induction. In *Topics* I.12 induction is exemplified by the following inference: “the skilled pilot is most effective” and “the skilled charioteer is most effective” therefore “in general the skilled man is best at his particular task” (105a12-16). The above inductive inference can be understood in two ways: either as an inference leading to a universal proposition from particular propositions or as an inference that justifies a given universal proposition by means of particular propositions. In the context of the *Topics*, where dialectical arguments are discussed, the second interpretation seems more pertinent. Dialectical arguments are aimed at conviction (*pistis*); a dialectical argument is employed in a debate, where two adversaries oppose one another and a

¹⁶ A similar analysis of the various meanings of induction in Aristotle’s philosophy is found in von Fritz, “Die *Epagoge* bei Aristoteles” (1971), pp. 642ff. and in Hess, “Erfahrung und Intuition bei Aristoteles” (1970), pp. 48-82.

¹⁷ von Fritz distinguishes dialectical induction and the induction dealt with in the *Posterior Analytics* II.19. He claims that the induction of the *Posterior Analytics* II.19, which can be made on the basis of a single case, is a sort of insight [“Die *Epagoge* bei Aristoteles” (1971), pp. 649-657]. Although I agree with von Fritz’ distinction, my interpretation of induction in the *Posterior Analytics* II.19 is different from his.

¹⁸ 42a3; 68b13; 32-37; 71a5-11.

dialectical problem is a subject, upon which the adversaries disagree. So in a dialectical context induction does not *form* a universal proposition by collecting particular propositions but establishes a universal *judgment*. Dialectical induction is an argument intended to convince an opponent by appealing to particular instances. Further examination of other references to induction supports the view that dialectical induction is indeed a method of justification.

In *Topics* VIII.2 Aristotle says that induction is most useful when addressing the crowd, whereas syllogism is usefully employed when addressing dialecticians (157a18-20). In this chapter he also discusses induction in the context of objections, claiming that only after one's opponent has made an induction can one raise an objection. Similarly, in *Physics* VIII.2 Aristotle says that it is not enough to state a proposition, but that one should also justify it by employing either inductive or demonstrative reasoning (252a24). These passages imply that dialectical induction is a means of persuasion, in which particular instances justify a universal statement. Hence, the role of dialectical induction is to lead one to assent to a given proposition by means of particular cases.

Turning to the *Prior Analytics* II.23, does Aristotle in this chapter introduce a different notion of induction? Here, he discusses induction from the perspective of syllogistic inference. From this viewpoint, Aristotle defines induction as an inference, in which the relation between the middle term and the major term is deduced through the minor term. For instance, given that "long-lived" is the major term A, "bileless" is the middle term B, and the various species of long-lived animals, such as man, horse, and mule, are the minor term C, an inference will be inductive when the predicative relation between A and B is proven through the particular species included in the minor term, C. The reduction of the inductive inference to the syllogistic form requires that the minor term and the middle term be equal in extension (i.e., C and B are convertible terms). That is, the minor term should include *all* bileless animals. This inference avoids the "inductive leap" that affects the validity of induction, since the universal conclusion is drawn from an enumeration of all cases.

Although dialectical induction, unlike the so-called "perfect induction", does not require an exhaustive enumeration of all cases, the latter is not essentially different from the former. In the above inductive inference, as well as in dialectical inductions, the inductive inference is not aimed at forming a proposition but at justifying it. The so-called "perfect induction" is argumentative in the sense that it yields a conviction as to the truth-value of an already given proposition. Furthermore, the convertibility requirement that renders the induction perfect does not affect Aristotle's evaluation of inductive inferences. By imposing the convertibility requirement, he avoids

both the logical and the methodological problem of induction. From Aristotle's point of view the enumeration of all cases, which renders induction logically valid, is doable. In specifying the reference of the minor term denoted by C, Aristotle enumerates species such as man, horse and mule, and not individual instances. Given that Aristotle considers the system of classification by genera and species finite, enumeration of all cases is methodologically possible.

In light of Ross's view, according to which perfect induction is a different type of inductive inference, one might expect this type of induction not to be considered an inferior type of argument. However, the status of induction as inferior to syllogism is not altered by the convertibility requirement. Concluding the *Prior Analytics* II.23, Aristotle says that induction is clearer to us, while syllogism is prior and more familiar by nature (68b35-37). Since enumeration of all cases has not changed the cognitive worth of induction, it seems incorrect to regard it as perfect induction. The notion of perfect induction presupposes the problem of induction. Aristotle, however, is troubled neither by the validity nor the certainty of induction. In his view syllogism and induction differ in their cognitive worth; the former uncovers the objective relations in reality, the latter is embedded in human perspective.¹⁹ The convertibility requirement that led Ross to propose the distinction between the induction discussed in the *Prior Analytic* II.23 and the dialectical induction of the *Topics* does not amount to an essential difference. Dialectical induction and the so-called "perfect induction" have the same two characteristics: both justify an already given proposition and both are clearer in relation to us as opposed to clearer by nature.

The argumentative notion of induction appears to be inapplicable to the induction discussed in the *Posterior Analytics* II.19. Apart from being discursive, a feature that does not accord with the instantaneous act of the *nous*, argumentative induction has two other features that do not fall into line with the *Posterior Analytics* II.19. First, argumentative induction proves propositions such as "all long-lived animals are bileless", whereas in the *Posterior Analytics* II.19 induction leads to universal concepts, such as "man" and "animal". Second, the argumentative induction does not form content but justifies an already given content. Being a means of justification, such an induction cannot serve as an adequate answer to the question raised in the *Posterior Analytics* II.19. In this chapter, Aristotle asks how we become acquainted with the universal concepts, whereas argumentative

¹⁹ I agree here with Engberg-Pedersen, "More on Aristotelian *Epagoge*" (1979), pp. 311-314 in claiming that Aristotle does not introduce the notion of "perfect induction" in this chapter.

induction presupposes the universal proposition as already given. In the *Posterior Analytics* II.19 Aristotle seems to employ the term *epagôgê* (induction) in a different sense. This sense, according to my interpretation can be found in the *Prior Analytics* II.21 and the *Posterior Analytics* I.1.

In his analysis of the notion of induction, Ross claims that in two passages Aristotle employs the word *epagôgê* in a non-technical sense. Both passages are in the *Analytics*, one in the *Prior Analytics* II.21 and the other in the *Posterior Analytics* I.1. With regard to these passages, Ross claims that the term *epagôgê* and the participle *epagomenos* refer to a *deductive* process, since in these passages Aristotle says that the particular becomes known by induction (67a23, 71a21).²⁰ Other commentators, such as Hamlyn and McKirahan have questioned this conclusion.²¹ They hold that induction is used in these passages in its technical sense. However, they both claim that in these passages “induction” is an application of a general principle to a particular case. Although they do not regard such an employment of the term “induction” as deductive they still consider it a way that leads from the universal to the particular. Induction, as interpreted by Hamlyn and McKirahan does not accord with the general characteristic of induction as a way from the particular to the universal. Next I show how induction in these passages accords with this basic characteristic of induction. I argue further that the *Posterior Analytics* II.19, the *Prior Analytics* II.21, and the *Posterior Analytics* I.1 all appeal to the same notion of induction.

1.5 INDUCTION IN THE POSTERIOR ANALYTICS I.1 AND THE PRIOR ANALYTICS II.21

Aristotle refers to induction in the *Posterior Analytics* I.1, when discussing the foreknowledge presupposed in demonstrative knowledge. Modifying the claim that the premisses of demonstration ought to be known in advance, Aristotle says:

But you can become familiar by being familiar earlier with some things but getting knowledge of the others at the very same time – i.e. of whatever happens to be under the universal of which you have knowledge. For that every triangle has angles equal to two right angles was already known; but that

²⁰ The same position is held by Engberg-Pedersen, “More on Aristotelian *Epagoge*” (1979), p. 305.

²¹ Hamlyn, “Aristotelian *Epagoge*” (1976), p. 170; McKirahan, “Aristotelian *Epagoge* in *Prior Analytics* 2.21 and *Posterior Analytics* 1.1” (1983), pp. 4-7.

there is a triangle in the semicircle here became familiar at the same time as the induction. (71a17-19)²²

Aristotle claims that although the premisses of demonstration should be known in advance, the minor premiss sometimes becomes known simultaneously with the conclusion. So with regard to the following first figure syllogism,

- (1) Every triangle has angles equal to two right angles;
- (2) The figure in the semicircle is a triangle;

Therefore: (3) the figure in the semicircle has angles equal to two right angles.

Aristotle argues that the conclusion (3) becomes known simultaneously with the induction, by which premiss (2) becomes known.

From the standpoint of the deductive interpretation of this passage, the phrase “of whatever happens to be under the universal” is crucial for understanding the notion of induction that is employed here. According to this interpretation, the knowledge of the universal precedes the knowledge of the particular and the inductive procedure is conceived of as an application of a universal concept to particular instances. By this view, knowing by induction that the figure in the semicircle is a triangle means that the universal concept “triangle” has been applied to the particular triangle, inscribed in the semicircle. This act of application yields the inference that leads to the conclusion that the sum of the angles of this triangle is equal to two right angles. Consequently, although the above inference involves induction, the deductive interpretation views it as comprising two deductive steps: (1) the application of a universal concept to a particular case, which forms the minor premiss; (2) the syllogistic inference, which leads to the conclusion.

However, Aristotle’s parenthetical gloss at lines 71a21-25 suggests that the phrase “whatever falls under the universal” does not describe the way by which the knowledge of premiss (2) is attained. Explaining his contention, Aristotle says:

For in some cases learning occurs in this way, and the minor term (*eschaton*) does not become familiar through the middle term (*meson*) –in cases dealing with what are in fact particulars and not said of any underlying subject. (71a21-25)

Analyzing the above syllogism into its terms, we get: “having angles equal to two right angles” as the major term; “triangle” as the middle term; and

²² *hama epagomenos egnōrisen* (became familiar at the same time as the induction).

“the figure in the semicircle” as the minor term. In the passage quoted above, Aristotle says that the minor term does not become known through the middle term. That is, the figure in the semicircle does not become known through the universal term “triangle” at all; therefore it certainly does not become known by application of a universal concept to a particular instance. Accordingly, in claiming that the figure in the semicircle is known inductively, Aristotle does not seem to refer to a deductive procedure in which a universal term is applied to particular instances.

An understanding of the inductive procedure as an advance from the particular to the universal seems to be at odds with Aristotle’s characterization of the object of induction. In the above quoted passage, Aristotle contends that induction is applied to the ultimate particular (i.e., “that which is not said of any underlying subject”).²³ This contention raises the question of how an advance from the particular to the universal can characterize a procedure by which one attains knowledge of a particular object. Aristotle’s discussion in *Metaphysics* VII.17 may provide an answer. There Aristotle views the substance as a cause. A search for a cause, according to this discussion, always takes the form of the question, “Why does one thing belong to another?” (1041a11). This mode of investigation is applied not only to such cases as “the moon is eclipsed” and “man is an animal”, in which the predicative relation is evident, but also to primary substances such as “house” and “man” (1041a33). In Aristotle’s view, predicative relations also subsist in the case of primary substances, although they are not stated explicitly. Questions such as “Why is a house, a house?” or “Why is a man, a man?” should be analyzed, according to Aristotle, into questions of the form “Why is this material a house?” or “Why is this body a man?” An inquiry into a primary substance ought to proceed from an analysis of the substance into its matter and form. In light of the analogy that Aristotle draws between questions of the form “Why is A B?” and questions of the form “Why is A A?” it may be concluded that the relation between matter and form corresponds to the relation between subject and predicate. According to this analysis, the primary substance, which is analogous to a grammatical subject, is regarded from a metaphysical point of view as a composite of subject and predicate.

In typical cases of predication, subject and predicate differ in their grammatical roles as well as in their extensions. Subject and predicate are related as a particular to a universal; the subject is usually narrower in extension than the predicate. Since Aristotle renders the relation between

²³ See also *Categories* V 2a11-15: “that which is neither said of a subject nor in a subject, e.g. this (*tode*) man or this horse”. Similarly, in the above-mentioned passage Aristotle refers to “this (*tode*) figure in the semicircle”.

matter and form in predicative terms, grasping the form embodied in matter is indeed an advance from the particular to the universal, namely the form common to several particulars. Understanding Aristotle's contention in the *Posterior Analytics* I.1 in light of his discussion of substances in *Metaphysics* VII.17, it may be concluded that the recognition that the figure inscribed in the semicircle is a triangle involves induction (i.e., an advance from the particular to the universal). The recognition that a certain figure is a triangle is a transition from seeing a configuration of lines (i.e., the material of which it consists) to the perception of a triangle (i.e., form).

Consequently, the term "induction" is not employed in the *Posterior Analytics* I.1 in an argumentative sense. The argumentative induction, which is discussed mainly in the *Topics*, is aimed at yielding conviction: the employment of an argumentative induction enables one to assign a truth-value to a given proposition. So making an argumentative induction does not involve a modification of content but a modification of the status given to it. By contrast, the induction discussed in the *Posterior Analytics* I.1 is not argumentative. Apart from the fact that it cannot be analyzed into premisses and a conclusion, it has no bearing on truth-values but is a cognitive process that yields a qualitative change in the grasped content. Described in modern terms, the inductive process discussed in the *Posterior Analytics* I.1 serves as a mediator between two cognitive states, sensation and perception. Prior to induction one has sensation: one *sees* a configuration of lines. Based on this raw material, induction leads to perception, namely to an apprehension of the form embodied in matter.²⁴

The same interpretation applies to the *Prior Analytics* II.21. Here Aristotle attempts to explain how, despite the logical necessity of syllogistic inferences, one may know the premisses and yet be ignorant of the conclusion. Although Aristotle never defends explicitly his theory of syllogism against the charge of *petitio principii*, raised first by Sextus Empiricus,²⁵ this chapter can be understood as providing such a defense. In order to answer the question of how knowledge of the premisses is compatible with an ignorance regarding the conclusion, Aristotle introduces the following first figure syllogism: "If A belongs to all B and B belongs to all C, then A belongs to all C". Since this inference is valid, the conclusion is implicitly included in the premisses; if one knows the premisses one potentially knows the conclusion. Yet Aristotle claims that a man who is ignorant of the existence of C will in fact fail to draw the conclusion "A

²⁴ Sensible forms are of two kinds: (1) qualities proper to each sense organ; (2) common sensibles like magnitude, number, motion, rest, and figure. The recognition of the figure in the semicircle involves a different type of perception, which I discuss in the sequel.

²⁵ Sextus Empiricus, *Pyrrhoneion Hypotyposeon* II, pp. 195-203.

belongs to all C". Aristotle attempts to explain a situation in which the proposition "A belongs to all B", despite its universality, does not necessarily imply the proposition "A belongs to all C", which is obviously a particular instance of the preceding proposition.

In accounting for this curious situation Aristotle introduces a distinction between two types of knowledge: universal knowledge and particular knowledge. He holds that the knowledge of the conclusion "A belongs to all C", which is implicitly included in the premiss "A belongs to all B", is merely universal knowledge. Having this universal knowledge, one can know in a sense the proposition "A belongs to C", without having a particular knowledge of it (67a17-19). As a result, knowing that A belongs to all B is compatible with ignorance of the conclusion "A belongs to all C", since one may still be ignorant of the existence of C (67a12-13).

The distinction between universal knowledge and particular knowledge sheds light on the role of induction in solving the problem, tackled by the *Prior Analytics* II.21. Since universal knowledge and particular ignorance are compatible, the particular knowledge of the conclusion cannot be derived from the universal knowledge of the premisses; it does not stem from an application of a universal proposition to a particular fact. Moreover, if the knowledge of C's being were actually a result of such an application, Aristotle would not be able to solve the problem that he raises in this chapter. For if C is known by means of the universal, how is it possible for one to know it universally and yet be ignorant of it particularly? The distinction between universal knowledge and particular knowledge suggests that the particular is known by means different from those involved in an application of the universal to the particular.

The relation between particular knowledge and induction becomes clear from Aristotle's criticism of Plato's response to Meno's puzzle:

The same thing holds for the argument in the *Meno*, that learning is recollection. For it never happens that one has foreknowledge of the particular, but simultaneously with the induction one receives knowledge of the particular, as if by an act of recognition. For we know some things directly, for example that the angles are equal to two right angles, if we see that the figure is a triangle. (67a21-25)

According to this passage, induction yields knowledge of the particular. Induction, then, is a means by which we know the particular; it is an act of recognition in which a configuration of lines is known to be a triangle. In this process the chronological priority pertains to the configuration of lines; from seeing it one is led to recognize it as a triangle. From the context in which the distinction between the two modes of knowledge appears,

knowledge of the universal concept “triangle” is not presupposed in the inductive process. In this passage, as well as in the *Posterior Analytics* I.1, induction is a process whereby the universal facet of the particular instance is grasped.²⁶ Consequently, Aristotle employs the term *epagôgê* in two senses: (1) an argumentative sense, in which a truth-value is assigned to a universal proposition by appealing to particular instances; (2) a cognitive sense, in which induction mediates between sensation and perception. My interpretation of *Posterior Analytics* II.19 shows that the term *epagôgê* is employed there in the cognitive sense.

1.6 INDUCTION IN THE POSTERIOR ANALYTICS II.19

One of the two questions that Aristotle answers in the *Posterior Analytics* II.19 is how the principles of knowledge become known. Modern commentators generally interpret this question as concerning a method of discovery. By this view, Aristotle’s description of the path from perception to experience *via* memory is understood as a genetic account of concepts acquisition.²⁷ However, Aristotle’s summary of his answer to this question implies that his concern here is not with methodology. After describing the gradual progress from perception to experience, Aristotle says:

Thus the states neither belong in us in a determinate form, nor come about from other states that are more cognitive; but they come about from perception ... (100a10-11)

In this passage Aristotle reiterates his answer to the question of how the first principles become known. It indicates that this question does not concern a method or a process of concept acquisition. Maintaining the correlation between Aristotle’s question and his answer, it seems that the question: “How do the first principles become known?” is to be construed as equivalent to the question of whether knowledge of the first principles is innate. Indeed, this question is phrased explicitly in the second paragraph of the *Posterior Analytics* II.19, where Aristotle asks “whether the states are not present in us but come about in us, or whether they are present in us but escape notice” (99b25-26).

Hence, the *Posterior Analytics* II.19 deals with the question of whether knowledge of the first principle is innate or acquired. In addressing this

²⁶ The same interpretation holds for the *Posterior Analytics* I.1 71a24-30.

²⁷ Barnes, *Aristotle’s Posterior Analytics* (1993), pp. 260-261; Hamlyn, “Aristotelian Epagoge” (1976), pp. 171-172; Leshner, “The Meaning of *Nous* in the *Posterior Analytics*” (1973), pp. 51-58.

question, Aristotle examines two alternatives: (1) there is no foreknowledge of the first principles; (2) there is full-fledged foreknowledge of the first principles. These two alternatives reflect, in fact, the argument concerning the impossibility of learning, put forward in Plato's *Meno*. In this dialogue Socrates restates Meno's doubt concerning the very possibility of acquiring knowledge, claiming that there is no learning of what is already known, while knowledge of the unknown cannot be acquired for we do not even know what to look for. What is known as "Plato's theory of recollection" is introduced as an answer to Meno's puzzle. In Socrates' view, learning is nothing but a recollection of full-fledged foreknowledge. This view accords with the second alternative that Aristotle presents: knowledge of the first principles is present in us but escapes notice. Aristotle does not reject Plato's approach altogether; he modifies it, claiming that knowledge of the first principles requires certain foreknowledge, but full-fledged knowledge of them is nevertheless acquired.

The *Posterior Analytics* II.19, like the *Posterior Analytics* I.1 and the *Prior Analytics* II.21, deals with the problem of learning, as put forward in Plato's *Meno*. Having rejected Plato's identification of learning with recollection, Aristotle introduces the notion of induction in accounting for the origin of universal concepts. In light of this analysis, the bearing of induction on the problem discussed in the *Posterior Analytics* II.19 is clear. The question of whether the knowledge of the first principles is innate is answered in the negative, while induction provides Aristotle's positive account for the origin of first principles.

Having established the role of induction in the *Posterior Analytics* II.19, we may go on to consider the question of the character of the inductive process itself. The *Posterior Analytics* II.19 leaves open the question of what the process called "induction" actually is. Acquisition of universal concepts is described as a gradual process, which starts with perception and ends in experience. On the other hand, perception is held to be the cognitive state that grasps universal concepts, or in Aristotle's words, "It is clear that it is necessary for us to become familiar with the primitives by induction, for perception too instills the universal in this way" (100b3-5).

Aristotle's contention that "perception is of the universal – e.g., of man and not of Callias the man" (100a17-b1) does not accord with his standard view, that perception is of particular objects (*On the Soul* 417b22 ff.). For instance, in the *Posterior Analytics* itself Aristotle clearly says that the universal cannot be perceived even if perception is of the such (*toioutde*) and not of the individual (87b28-31). Further, according to *On the Soul* II.6, the proper objects of perception are either qualities unique to a particular sense or the common sensibles such as motion, rest, number, figure, and

magnitude. This discrepancy can be settled in light of a distinction between two senses of *aisthesis*, made in the *Nicomachean Ethics* VI.8:

...practical wisdom is concerned with the ultimate particular, which is the object not of knowledge but of *aisthesis* – not *aisthesis* of qualities peculiar to each sense but an *aisthesis* akin to that by which we perceive that the particular figure before us is a triangle. (1142a26-29)

Recognition of a certain configuration of lines as a triangle, which is an induction according to the proposed interpretation, presupposes a different sense of perception from that discussed in *On the Soul* II. Perception in the inductive sense grasps the universal; it grasps the form, which is embodied in matter. Since the form of a particular instance is common to all of the instances that belong to a certain species, grasping a triangle or any other particular object involves a generalization. Accordingly, a perception of a particular definite object is an apprehension of the universal facet, i.e., the form, of this object. A comparison of the *Posterior Analytics* II.19 with the parallel passage from *Metaphysics* I.1 indicates that in the former the term “induction” connotes perception in same sense as it appears in the *Nicomachean Ethics* VI.8, while the gradual process from perception to experience, taken by itself, is not inductive.

In *Metaphysics* I.1, Aristotle says:

And from memory experience is produced in men; for many memories of the same thing produce finally the capacity for a single experience. Experience seems to be very similar to knowledge and skill, but really knowledge and skill come to men through experience. (980b28-981a5)

According to this passage, experience is the basis of knowledge and skill, but it is not identical with knowledge and skill themselves. The example given in *Metaphysics* I.1 reinforces this contention. Exemplifying the process of learning, Aristotle distinguishes experience and universal judgment; experience consists of many particular judgments concerning Callias, Socrates, etc., whereas a universal judgment concerns all men. Hence, experience is not a generalization of items preserved in memory but a collection of these memories. Aristotle claims that experience is of individuals (981a9-10); therefore experience, which is the final stage of the process, leading to art and reasoning, is still rooted in perception.

According to *Metaphysics* I.1, experience does not involve a qualitative modification of the material collected by perception and memory. Perception and memory as well as experience gather individuals. Similarly, in the *Posterior Analytics* II.19 Aristotle does not claim that a qualitative

modification of the perceived content occurs at the stage of experience. This process is described as beginning with a universal term, i.e., a certain species, such as “man”, and ending with another universal term, i.e., a certain genus, such as “animal”. Unlike the process described in the *Posterior Analytics* II.19, *Metaphysics* I.1 does not describe a process involving a transition from the particular to the universal. By contrast, the inductive process of the *Posterior Analytics* II.19 opens with universal concepts, i.e., with species, which are grasped by perception (100a15-b3). The process described in the *Posterior Analytics* II.19 employs induction at the very first stage of perception. This contention finds support in Aristotle’s concluding remark at the end of his discussion of the acquisition of the first principles: “thus it is clear that it is necessary for us to become familiar with the first principles by induction; for perception too instills the universal in this way” (100b3-5). Induction, then, is the act of recognizing the universal facet of the particular by perception.

In *Metaphysics* I.1, as well as in the *Posterior Analytics* II.19, the process that begins with perception and ends with experience does not lead directly to the first principles but provides the raw material for formulating the first principles.²⁸ In *Metaphysics* I.1 Aristotle maintains that a universal judgment is formed through experience, and in the *Posterior Analytics* II.19 the principle of knowledge emerges from experience. In *Metaphysics* I.1 the act of generalization is not part of the process that leads from perception to experience, whereas in the *Posterior Analytics* II.19 the progress from one stage to another involves generalization. Accordingly, the parallelism between *Metaphysics* I.1 and the *Posterior Analytics* II.19 holds only for the characteristics of the stages involved in the process of learning, i.e., perception, memory, and experience. *Metaphysics* I.1, unlike the *Posterior Analytics* II.19, does not describe an inductive process. The process described in *Metaphysics* I.1 is one of collecting particular instances, while that described in the *Posterior Analytics* II.19 is inductive, since its very first stage involves a generalization.²⁹

Consequently, the notion of induction that is employed in the *Posterior Analytics* II.19 is the same notion employed in the *Prior Analytics* II.21 and the *Posterior Analytics* I.1. In these three passages induction is a perceptual

²⁸ In the *Posterior Analytics* II.19 100a5-7 Aristotle says explicitly that the first principles stem from experience. In the *Posterior Analytics* I.2 Aristotle classifies definitions under posits (*thesis*). Hence, definitions have to be laid down and they are not apprehended immediately by the act of the intellect. Similarly, in *Metaphysics* I.1 the principles of knowledge and skill have to be posited by an act of universal judgment.

²⁹ This interpretation explains the fact that the term *epagôgê* does not appear in *Metaphysics* I.1. A distinction between induction and the process that leads from perception to experience is endorsed also in Hamlyn, “Aristotelian Epagoge” (1976), p. 182.

act; it is a transition from grasping the matter to an apprehension of the form. Further, the comparison of the *Posterior Analytics* II.19 with *Metaphysics* I.1 leads to the conclusion that induction does not occur during the entire process, which starts with perception and ends in experience. This process is inductive from the outset; its first stage requires an inductive act, by which the universal concept “man” is educed from a particular man, e.g., Callias.

When induction is viewed as a process of grasping the form, the correlation between induction and intellect becomes straightforward. In this interpretation induction is not an argumentative procedure but an immediate act in which the universal is apprehended. Moreover, in this interpretation the notion of intellect, introduced in the *Posterior Analytics* II.19, is compatible with Aristotle’s discussion of the intellectual capacity in *On the Soul* III.³⁰ In *On the Soul* III.4, Aristotle says:

Since we can distinguish between a magnitude and what it is to be a magnitude, and between water and what it is to be water, and so in many other cases (though not in all) ... flesh and what it is to be flesh are discriminate either by different faculties, or by the same faculty in two different states; for flesh necessarily involves matter and is like what is snub-nosed, a this in a this. Now it is by means of the sensitive faculty that we discriminate the hot and the cold, i.e. the factors which combined in a certain ratio constitute flesh: the essential character of flesh is apprehended by something different either wholly separate from the sensitive faculty or related to it as a bent line to the same line when it has been straightened out. (429b10-18)

In this passage Aristotle asks whether the same faculty discriminates the object and its essence. Since flesh and water are both matter, and magnitude is a form embodied in matter, Aristotle’s question seems odd. Does Aristotle seriously consider the possibility that the objects of sensation and the objects of intellection are grasped by the same faculty? The solution to this question lies in Aristotle’s claim that flesh is “a this in a this”. “Flesh” is not considered here a material component of a compound object like the human body, but is considered the compound object itself. Flesh consists of matter (the hot and the cold) and form, i.e., a certain ratio. Since flesh has essence, Aristotle asks what is the faculty that grasps this essence. This faculty cannot be sensation for sensation grasps the hot and the cold; therefore the intellectual faculty could be the appropriate candidate for accomplishing this

³⁰ Leshner casts doubt on the possibility of drawing a correspondence between the concept of *nous* employed in the *Posterior Analytics* and that discussed in *On the Soul* III. [Leshner, “The Meaning of *Nous* in the *Posterior Analytics*” (1973), p. 46]

task. If this interpretation is correct, grasping the form of a particular object is, according to Aristotle, an intellective act. Intellect is the faculty of grasping forms, while induction is the means of grasping them. Therefore the identification of the intellect with the state by which first principles are attained is compatible with induction.

1.7 EPISTEMOLOGICAL IMPLICATIONS

In his commentary on the *Posterior Analytics* II.19, Barnes puts forward three questions that affect the understanding of this chapter as a whole: (1) Whether Aristotle adopts an empiricist or a rationalistic attitude towards the process of attaining the first principles. (2) Whether the first principles are concepts or propositions. (3) The relation between the last chapter of the *Posterior Analytics* and the rest of the second book. The answers to the first two questions disclose the difference between Aristotle's theory of knowledge and the modern conceptions of knowledge.

The various answers given to the first question portray Aristotle either as an empiricist or as a rationalist. Barnes follows the first approach while Kahn takes the second.³¹ Both commentators understand the relation between induction and intellect as analogous to the relation between demonstration and knowledge. Although the interpretation proposed here follows the same lines, it implies that the terms empiricism and rationalism are rather misleading. In the *Posterior Analytics* II.19 Aristotle does not suggest a Baconian notion of induction, according to which induction is a method of generalization that leads from the observation and collection of facts to certain general statements. In Aristotle's view, induction is not identical with the process that starts with sense perception and ends with experience. Induction, according to Aristotle, is an immediate apprehension of essences or forms, occurring at the first stage of the path from sense perception to experience. Such an apprehension does not presuppose a collection of particular instances, since perception and memory already involve an acquaintance with universal concepts. On the other hand, intellection is not a rationalist act, as Kahn claims. Aristotle views induction as a transition from sensation to perception, i.e., from matter to form. Therefore, sensual acquaintance with a particular instance is indispensable for the process of induction. This interpretation undermines Kahn's rationalist interpretation, according to which Aristotle's discussion of the

³¹ Barnes, *Aristotle's Posterior Analytics* (1993), pp. 261-267; 270; Kahn, "The Role of Nous in the Cognition of First Principles" (1981), p. 411.

acquisition of first principles is based on the assumption that the intellect is indifferent to our personal experience.³²

The second question concerns the discrepancy between the conceptual and the propositional accounts of the first principles. Kahn attempts to reconcile this discrepancy by reducing all principles of knowledge to concepts and corresponding existence claims. Both types of principles, i.e., concepts and existence claims, are grasped in one and the same inductive process, for definitions and existence claims are not separate propositions. According to Kahn, definitions are universal affirmative propositions of the form “all A are B”, which implicitly state that A exists. However, Kahn’s interpretation cannot explain the significance of several discussions throughout the *Posterior Analytics*, where Aristotle states that demonstrative premisses can be atomic negative propositions (I.15); furthermore Aristotle’s argument for finite negative chains of predication (I.21) and his claim that an affirmative demonstration is better than a negative one (I.25) become redundant under Kahn’s interpretation. Ross and Barnes, on the other hand, do not give a clear answer to this question. Ross argues that the formation of universal concepts and the grasp of universal propositions are “inseparably interwoven”; although the two processes are distinct, they are alike in being inductive.³³ Barnes explicitly leaves this question unanswered, because in his opinion Aristotle vacillates between a propositional account and a conceptual account of the first principles.

These two questions cannot be answered satisfactorily, since they stem from modern presuppositions. The considerations that led Aristotle to pose the question of the acquisition of first principles are different from the considerations that led seventeenth-century philosophers to appeal to induction and self-evident truths. Induction, for Aristotle, is not a means of establishing knowledge, i.e., its objective is not justification. Induction is introduced by Aristotle in order to solve a Greek problem, put forward in Plato’s *Meno*. Meno’s puzzle concerns explaining learning as a process, and not establishing solid foundations for knowledge. By contrast, empiricism and rationalism are two modern theories about the sources of knowledge; the dispute between them centers on whether experience or reason provides the genuine foundation of knowledge. Since Aristotle was not troubled with this question, relating *epagôgê* to an empiricist approach and *nous* to a rationalist approach is anachronistic and misleading.

³² The difficulties in phrasing Aristotle’s conception in terms of empiricism/rationalism find expression in Ross’s claim that the intellectual capacity has both features: Ross, *Aristotle* (1977), p. 39. This approach can also be found in Lee, “Geometrical Method and Aristotle’s Account of First Principles” (1935), p. 122, and Le Blond, *Logique et méthode chez Aristote* (1970), pp. 131-132.

³³ Ross, *Aristotle’s Prior and Posterior Analytics* (1949), p.675.

The difference between Aristotle's theory of knowledge and modern epistemology finds expression in the answer to the second question. Reading into the text the notion of *nous* as the intuitive apprehension of immediate propositions stems from the modern preoccupation with skeptical objections to epistemology. Modern epistemology looks for self-evident propositions, which can serve as the cornerstone of knowledge. Self-evident propositions are intuited as true, namely they are fundamental judgments upon which all other judgments are based. Aristotle, by contrast, treats intellect as a means of understanding, i.e., the intellect grasps essences and not judgments. Since intellect is not a judgmental capacity, it does not in itself guarantee the truth of any derived propositions. Therefore it does not serve as the epistemic foundation of an axiomatic system. Neither the question of rationalism and empiricism nor the conceptual *versus* propositional accounts of the first principles is an Aristotelian question; both arose from reading modern notions into Aristotle's account.³⁴

The relation between the last chapter of the *Posterior Analytics* and the rest of the book is treated in my next chapter. To pave the way to the forthcoming discussion I would like to reject one of the answers that has been given to this question. In his commentary on the *Posterior Analytics*, Barnes argues that Aristotle wrote chapter II.19 as an independent essay, which he attached to the second book of the *Posterior Analytics* at a later stage. Therefore Barnes sees this chapter as the generalization of a thesis stated throughout the second book, especially in chapter II.13, which deals with the acquisition of definitions.³⁵ An elaborated justification of this view is proposed by Kosman, who identifies the process of demonstration with the grasp of the essence by the intellect.³⁶ According to Kosman, demonstration and induction are in fact one and the same scientific activity, discussed from different aspects. Insight and induction are employed in every demonstration, which has an explanatory role and reveals the essence.

Aristotle does indeed claim that demonstration is a means of revealing an essence; however, the essence revealed in demonstration is not the essence acquired by the intellect. In the *Posterior Analytics* 1.31 Aristotle says:

³⁴ Aristotle's ancient and renaissance commentators, such as Philoponus and Zabarella, raised neither the first question nor the second question.

³⁵ In his comments on Mario Mignucci's commentary on the *Posterior Analytics*, Goldschmidt claims that II.19 is an early draft of the *Posterior Analytics* II.1-18. In his article "L'object et la structure des *Seconds Analytiques* d'après Aristote" (1981), pp. 61-96, Brunschwig criticizes this assumption. By examining the structure of the *Posterior Analytics*, he argues that the question of the acquisition of the first principles is not been treated until the end of the book. Brunschwig's criticism applies to Barnes's position as well.

³⁶ Kosman, "Understanding, Explanation and Insight in the *Posterior Analytics*" (1973).

The universal is valuable because it makes clear the cause; hence universal demonstration is more valuable than perception and intellection (*noesis*) – with regard to those things whose cause is something different; but for the primitives there is a different account. (88a5-8)

Here Aristotle distinguishes things whose cause is different from primitives. According to the *Posterior Analytics* I.10, the genus or the subject of the conclusion is a “primitive” (*to prôton*). But in the *Posterior Analytics* II.8 Aristotle says that the essence revealed by demonstration is the essence of the predicate term. For example, a demonstration that reveals why the moon is eclipsed actually answers the question “What is an eclipse?” A demonstration, then, reveals the essence of the predicate term “eclipse”; the essence of the subject term “moon” does not become clear *via* demonstration. In chapter II.9 Aristotle says that this account holds for things whose cause is different; for other things, the essence is a principle. Aristotle distinguishes between the essence that is a principle of demonstration and the essence that is revealed in demonstration; the former is the essence of the conclusion’s subject term, i.e., the genus, while the latter is the essence of the conclusion’s predicate term, i.e., its attribute. My alternative account of the role of the intellect in demonstration will become clear in the following discussion of the modal status of the second type of *archê*, which is the immediate premiss.

Chapter Two

The Immediate Premiss

2.1 IMMEDIATE PREMISES AND SKEPTICISM

The claim that demonstrative knowledge requires non-demonstrable first principles is stated in the *Posterior Analytics* I.3 as an alternative to two skeptical dismissals of the very possibility of attaining knowledge. Aristotle's response to the first skeptical objection is important for understanding the relation between the two types of first principles: the intellect and the immediate premiss. The first skeptical argument is based on the assumption that demonstration is the only means by which knowledge can be attained. Attaining demonstrative knowledge requires that any proposition that serves as a demonstrative premiss must be proven in advance. But on this assumption, the attainment of knowledge becomes impossible, since the chain of proved propositions will continue *ad infinitum*, and could never be exhausted (72b5-11). The skeptic who poses this objection maintains further that even if the chain of propositions were finite there would be no genuine knowledge, since the ultimate proposition has not been proven. In that case, the knowledge that follows from these unproven propositions is not knowledge *simpliciter*, but a hypothetical knowledge, conditioned by the premisses (72b14).

Aristotle rejects this skeptical view:

But we say that neither is all knowledge demonstrative, but in the case of the immediates it is non-demonstrable...and we *also* (*alla kai*) say that there is not only knowledge but also some principle of knowledge by which we become familiar with the definitions. (72b18-25)

According to this passage, Aristotle's alternative to the skeptical view includes two theses, distinguished, in the Greek text, by the particle *alla kai*: (1) there is non-demonstrable knowledge; (2) there is a principle of knowledge, by which definitions are grasped. It is only the conjunction of these two theses that guarantees the rejection of the skeptical view. For while non-demonstrable knowledge does answer the infinite regress claim, it is insufficient as grounds upon which the contention that all knowledge is

hypothetical can be rejected. A non-demonstrable premiss, even if it is an ultimate premiss, is nevertheless accepted without proof, therefore the knowledge based on such a premiss is conditioned and hypothetical. If Aristotle were rejecting the skeptical view only on the basis of non-demonstrable knowledge, his position would be identical with the skeptical position. The second part of Aristotle's answer to the skeptical view is aimed at avoiding this consequence. Truth is not the only mark of the immediate premiss; Aristotle requires further that this truthful immediate premiss be assumed on the basis of a first principle (*dia tôn ex archês*) (*Prior Analytics* 24a30). In this chapter, I show that this contention prevents Aristotle's demonstrative knowledge from being hypothetical knowledge. By analyzing the relations between definitions and hypotheses, I argue that definitions as principles of knowledge guarantee the unconditioned epistemological status of the immediate proposition. In order to determine this role of definitions in forming the immediate premisses, I first discuss Aristotle's distinction between definitions and hypotheses.

2.2 DEFINITIONS AND HYPOTHESES

In the *Posterior Analytics* I.2, Aristotle distinguishes three types of syllogistic principles:

An immediate syllogistic principle I call a thesis if one cannot prove it, but it is not necessary for everyone who is to learn anything to grasp it; and one which it is necessary for anyone who is going to learn anything whatever to grasp, I call an axiom (for there are such things); for we are accustomed to use this name especially of such things. A thesis which assumes either of the parts of a contradiction – i.e., I mean, to be something or not to be something – I call a hypothesis; one without this, a definition. For a definition is a thesis (for the arithmetician posits that a unit is what is quantitatively indivisible) but not a hypothesis (for what is a unit and that a unit is are not the same). (72a14-24)¹

Aristotle's classification of first principles opens with a distinction between axioms and theses. According to this classification axioms are first principles, required for any learning whatsoever, whereas theses are required

¹ Barnes's translation is revised here. Barnes renders the phrase "*to einai ti*" as "something exists". Literally, it means "to be something". Barnes's translation, then, assumes the existential interpretation of hypotheses.

for learning specific branches of knowledge. Aristotle does not elaborate on the role of axioms in demonstration, but his claim in the *Posterior Analytics* I.11 that the principle of the excluded middle need not be explicitly assumed in demonstration indicates that the axioms, though first principles, do not serve as premisses in demonstrations. The other class of first principle – the theses – is further divided into hypotheses and definitions. The former is an assumption of either part of a contradiction, while the latter is characterized as “without this” (*aneu toutou*). According to the previous paragraph being one part of a contradiction is a mark of immediate premisses (72a7-9), so among the first principles listed in this passage only hypotheses seem to serve as premisses in syllogistic demonstrations.

Unlike the distinction between axioms and theses that seems to be unambiguous, Aristotle’s distinction between definitions and hypotheses can be interpreted in two different ways. From his examples, the distinction between definitions and hypotheses can be construed as one between predications and existence claims. Aristotle exemplifies definitions by the expression “what a unit is” and hypotheses by the expression “that a unit is”. Moreover, his example of definitions – “a unit is what is quantitatively indivisible” – may support the existential interpretation of hypotheses. For if both definitions and hypotheses are predicative propositions, Aristotle’s sharp distinction between the two would be rendered indistinct. On these grounds, it may be argued that the Greek phrase *to einai ti* means “something exists”; by this reading, hypotheses are viewed as existential propositions and definitions as predicative propositions, lacking an existential import.²

However, in the *Posterior Analytics* I.10 Aristotle maintains that hypotheses, unlike definitions, serve as premisses in a syllogistic demonstration (76b35-39). Syllogistic inferences proceed from two predicative propositions, relating two different terms to a third term, i.e., the middle term. Hence, since a one-term proposition, such as “*x* exists”, cannot

² The existential interpretation of hypotheses is held by the majority of commentators such as: *Sancti Thomae Aquinatis, In Aristotelis Libros Peri hermeneias et Posteriorum Analyticorum Expositio*, (1975), L I, I, Lectio V, n.51 (8); *Themistii in Analyticorum Posteriorum Paraphrasis*, (1900), 7. 10-24; Barnes, *Aristotle’s Posterior Analytics* (1993), pp. 100-101; von Fritz, “Die Archai in der griechischen Mathematik” (1971), p. 390; Goldin, *Explaining an Eclipse: Aristotle’s Posterior Analytics 2. 1-10* (1996), pp. 41-51; Heath, *Mathematics in Aristotle* (1970), pp. 53-57; Landor, “Definitions and Hypotheses in *Posterior Analytics* 72a19-25” (1981), pp. 308-318; Lee, “Geometrical Method and Aristotle’s account of First Principles” (1935), pp. 113-124; Mansion, *Le Jugement d’existence chez Aristote* (1976), p. 153; McKirahan, *Principles and Proofs – Aristotle’s Theory of Demonstrative Science* (1992), pp. 122-132; Ross, *Aristotle’s Prior and Posterior Analytics* (1949), p. 57; Szabo, *Anfänge des euklidischen Axiomensystems* (1960), p. 105.

serve as a syllogistic premiss, an existential interpretation of hypotheses fails to account for their role in syllogistic inferences. Demonstration is a type of syllogistic inference; therefore the existential interpretation poses a fundamental problem for a comprehensive interpretation the *Posterior Analytics* as a whole.

These reasons led Gomez-Lobo to offer an alternative interpretation for the distinction between definitions and hypotheses.³ He sets for it two objectives: (1) it should accommodate Aristotle's notion of hypotheses with the syllogistic form of inference; (2) it should maintain the parallelism between Aristotle's classification of the first principles and the structure of Euclid's *Elements*. In light of these objectives, hypotheses are construed as singular propositions, having the form "This is an F".⁴ According to Gomez-Lobo, Aristotle's hypotheses correspond to the stage of a Euclidean proof, which Proclus calls *ekthesis*, and not to Euclid's postulates, which are traditionally understood as implying existence. Still, Gomez-Lobo's interpretation fails to accommodate hypotheses to the syllogistic form of demonstration. Apart from the fact that singular propositions cannot serve as syllogistic premisses, in the *Prior Analytics* I.27 Aristotle claims that singular terms are not included among syllogistic terms (43a25-43). Moreover, in none of Aristotle's examples of demonstrations does an ostensive proposition of the form "This is an F" occur. So both interpretations, the existential and the singular, fail to explain the classification of hypotheses under the category of syllogistic premisses.

Aristotle's characterization of hypotheses follows his definition of the term "premiss" (*protasis*). In this context Aristotle says that a premiss is "one part of a contradiction" (*apophanseôs to heteron morion*) (72a8-14); to clarify he says that in a premiss "one thing is said of one thing" (*hen kath' henos*) (72a9). Aristotle's characterization of a premiss as "one part of a contradiction" indicates that a premiss is a predicative proposition, in which

³ Gomez-Lobo (1977).

⁴ Another emendation of the existential interpretation has been suggested by Hintikka, "On the Ingredients of an Aristotelian Science" (1972), pp. 63-64. Hintikka claims that hypotheses are not existential propositions of the form "x exists" but are predicative propositions that include a hidden existential assumption, to the effect that the subject of predication exists. On the other hand, Owens, *The Doctrine of Being in the Aristotelian Metaphysics* (1978), pp. 289-291 draws a distinction between the notion of existence that refers to contingent attributes of a certain entity and the notion of being that refers to universal and necessary connection between the elements of definitions. He claims, on the basis of this distinction, that Aristotle's concerns here is with being as opposed to existence.

one thing is said of another: a premiss is a proposition of the form “A is B”.⁵ Further, the expression “one part of a contradiction” implies that a hypothesis is either an affirmative proposition or its contradictory negative proposition. The existential interpretation as well as the singular interpretation should then explain the role of statements of the form “This is not an F” or “F does not exist” in a deductive science.

Moreover, Aristotle’s claim that hypotheses may be negative propositions undermines Gomez-Lobo’s parallelism between Aristotle’s hypotheses and Euclid’s *ekthesis* stage. The latter is the act of setting out a particular instance that corresponds to a universal term. For example, given the universal proposition “On a given straight line to construct an equilateral triangle” (*Elements* I.1), the proposition “Let AB be the given straight line” is laid down at the *ekthesis* stage. Hence, the propositions laid down at the *ekthesis* stage instantiate a universal term, so only singular affirmative propositions can appear at this stage; but Aristotle’s hypotheses can be either affirmative or negative.

Furthermore, neither the existential interpretation nor the singular interpretation has much support in textual evidence. The main evidence introduced in supporting the existential interpretation of hypotheses is in the *Posterior Analytics* I.10, where Aristotle requires the assumption of existence for the primitive terms of a demonstrative branch of knowledge. Yet, how this requirement bears on the interpretation of hypotheses is unclear as it applies to the conclusion’s subject term, while the hypotheses, discussed in *Posterior Analytics* I.2, serve as the premisses of demonstration.⁶ On the other hand, there is textual evidence to support the predicative view: In the *Posterior Analytics* I.19 Aristotle says:

Every deduction is through three terms; and one type is capable of proving that A belongs to C because it belongs to B and that to C, while the other is negative ... So it is evident that the principles and *what are called* hypotheses are these; for it is necessary to assume this and prove in this way – e.g. that A belongs to C through B ... (81b10-16)

⁵ Among the advocates of the non-existential interpretation are Barnes, *Aristotle’s Posterior Analytics* (1975), pp. 103-104; Robinson, *Plato’s Earlier Dialectic* (1953), p. 101.

⁶ In the previous chapter I made a similar claim regarding the distinction between the principles of demonstration, which are the premisses of demonstration and the principle of knowledge, which is the genus or the conclusion’s subject term. The question *ei esti* discussed in the *Posterior Analytics* II. 1-10 is irrelevant to Aristotle’s notion of hypotheses for the same reason. The assumptions of the genus’s being and the signification of its attributes are discussed in chapter five.

In attempting to explain away predicative examples of hypotheses, the advocates of the existential interpretation distinguish the technical and the non-technical meanings of the term “hypothesis”. A hypothesis in the technical sense is an existential proposition that serves as a premiss in demonstration. The non-technical sense of “hypothesis” is employed in places such as *Physics* 185a12 and 253b5, where a predicative proposition, such as “Nature is a source of change”, is said to be a hypothesis. Yet the passage quoted above seems to undermine this construal of Aristotle’s notion of hypothesis. In this passage, the term “hypothesis” refers unambiguously to predications, i.e., to propositions having the form: “A belongs to B”. Furthermore, the expression *ai legomenai hupotheseis* (i.e., “what are called hypotheses”) indicates that here Aristotle is referring to a technical sense of the term hypothesis. This passage implies that within the context of the *Posterior Analytics* the term “hypothesis” in its technical sense refers to predicative propositions.

However, an endorsement of the predicative interpretation of hypothesis intensifies the problem of differentiating definitions and hypotheses. Given that both are predicative propositions, it is not clear what feature exactly distinguishes them. If hypotheses as well as definitions are propositions of the form “A is B”, on what basis can we classify one proposition as a definition and another as a hypothesis? The following discussion of the expression “one part of a contradiction” and its relation to being provides an answer.

The problem in Aristotle’s characterization of hypotheses is the discrepancy between their predicative form and their existential import. In the *Posterior Analytics* I.2 Aristotle characterizes hypotheses as “one part of a contradiction”, which according to him means “to be something” (*to einai ti*) or “not to be something” (*to mê einai ti*). In *Metaphysics* VI.4 Aristotle uses the expression “one part of a contradiction” in introducing one of the meanings of being, called “being as true” (*to hos alêthes on*):⁷

But since that which *is* in the sense of being true, or *is not* in the sense of being false, depends on combination and separation, and truth and falsehood together are concerned with the apportionment of a contradiction (*peri merismon antiphaseôs*) (for truth has the affirmation in the case of what is compounded and the negation in the case of what is divided, while falsity has the contradictory of this apportionment ...) (1027b18-23)

⁷ “Being as true” is one of the four meanings of the term “being” that Aristotle discusses in *Metaphysics* V.7 1017a31-35. None of these meanings is existential in the sense implied by propositions of the form “X exists”.

Interpreting Aristotle's notion of hypothesis in light of this passage settles the discrepancy between the predicative form of hypotheses and their existential import. Having being characterized as "one part of a contradiction", hypotheses state "*to einai ti*", in the sense of "being either true or false". In this interpretation, hypotheses are predicative propositions that in being affirmations or negations imply being in its veridical sense. Rather than being existential propositions of the form "X exists", hypotheses are predicative propositions that tacitly state, "It is the case that A is B" or "It is not the case that A is B".

Further, an interpretation of Aristotle's notion of hypotheses in light of his discussion of "being as being true" may clarify the distinction between definitions and hypotheses. In *Metaphysics* VI.4 Aristotle draws a distinction between composites and in-composites, claiming that truth and falsity refer to them in different ways. Composites, which are affirmations and negations, can be either true or false, whereas in-composites, which are associated with essences (*ta hapla kai ta ti esti*), cannot be false (1027b27-28). In *Metaphysics* IX.10 Aristotle elaborates on this:

In fact, as truth is not the same in these cases, so also being is not the same; but truth or falsity is as follows – contact and assertion are truth (assertion not being the same as affirmation), and ignorance is non-contact. For it is impossible to be in error regarding the question *ti esti*. (1051b22-26)

Accordingly, definitions that answer the question "what it is" (*ti esti*) differ from propositions in having only one truth-value, i.e., the true. Aristotle's distinction between an affirmation (*kataphasis*) and an assertion (*phasis*) indicates that the difference between definitions and propositions does not lie in their sentential form.⁸ Affirmations as well as assertions are predicative propositions; the difference between them lies in the truth-value assigned to

⁸ In discussing the relation between the in-composites and truth-value, Aristotle refers to them as mental acts of grasping a certain content. Yet, in *Metaphysics* IX.10, *Metaphysics* VI.4, and *On the Soul* III.6 Aristotle contrasts in-composites with composites, viewing the latter as propositions. The following consideration supports the assumption that Aristotle's characterization of the relation between the in-composites and truth-value is applicable to the sentential representation of the mental acts that grasp these in-composites (i.e., definitions). In *Metaphysics* IX.10 Aristotle analyses the distinction between composites and in-composites in terms of the distinction between affirmation (*kataphasis*) and assertion (*phasis*). In *De Interpretatione* I.4 Aristotle distinguishes affirmations from assertions saying that a part of a sentence, i.e., a single word that conveys meaning, is an assertion but not an affirmation (16b26-28). Given that for Aristotle an affirmation involves one thing being said of another, and given that definitions do not involve combination and separation (*Met.* VII.12), it seems reasonable to conclude that definitions (as sentential entities) fall under the category of *phasis*, despite their predicative form.

them. As opposed to assertions, affirmations are propositions that have an alternative: they can be true or false. Definitions, by contrast, are assertions: they have no intelligible alternative and therefore they can have only one truth-value, which is the true. By distinguishing between hypotheses and definitions in terms of being, Aristotle thus does not refer to being in the existential sense but in the sense of being true. Hypotheses imply being since they are either true or false; definitions, by contrast, lack this import since they cannot be false.

The existential interpretation of “hypothesis” stems from an attempt to explain the distinction between definitions and hypotheses in terms of their sentential form. In this view, the characteristic “one part of a contradiction” cannot distinguish definitions from hypotheses since the advocates of the existential interpretation conceive of the expression “one part of a contradiction” as characterizing any proposition, including definitions.⁹ However, from the foregoing analysis it may be concluded that the distinction between definitions and hypotheses is not determined on formal grounds. According to Aristotle, definitions as well as hypotheses are predicative propositions. Yet unlike hypotheses, definitions are *not* an apportionment of a contradiction. The expression “one part of a contradiction” characterizes predicative propositions according to the relation they have or do not have to a contradictory proposition. Hence, the difference between hypotheses and definitions is the difference between propositions that may bear one of the two alternative truth-values (hypotheses) and propositions that have no alternative truth-value (definitions).¹⁰ The implications of this interpretation for the modal status of hypotheses are considered in the following section.

⁹ Landor, “Definitions and Hypotheses in the *Posterior Analytics* 72a19-25 and 76b35-77a4” (1981), p. 309; Barnes, *Aristotle’s Posterior Analytics* (1993), p. 100.

¹⁰ By contrast, the difference between the question *ei esti* and *ti esti* is not as sharp as the difference between these two modes of being. In *Metaphysics* VI.1 (1025b17-18) Aristotle claims that the questions *ti esti* and *ei esti* belong to the same line of thought (*to tês autês einai dianoias to te ti esti dêlon poiein kai ei esti*). Furthermore, in the *Physics*, where Aristotle often distinguishes these two questions, the distinction is never clearly set out. For example, the question of whether time exists is answered by asserting that “time is such and such”. In this as well as in other discussions in the *Physics* it is often hard to decide when Aristotle is answering the question of being and when he is indicating the essence. The distinction between these two questions is discussed in chapter 5.

2.3 THE PROBLEM OF THE MODAL STATUS OF HYPOTHESES

In the *Posterior Analytics* I.10 Aristotle distinguishes between hypotheses and postulates: a hypothesis is an unproved proposition that a learner accepts to be true whereas a postulate is either a proposition that the learner considers to be false or a proposition of which the learner has no opinion (76b23-31). Thus the assignment of a truth-value to a proposition serves as a distinguishing mark between hypotheses and postulates.¹¹ A proposition is classified either as a hypothesis or as a postulate according to the learner's judgment.

This distinction between hypotheses and postulates assumes a further distinction between hypotheses *simpliciter* and relative hypotheses, which can be understood in two ways. (1) A relative hypothesis is a provable proposition that has been stated without proof while a hypothesis *simpliciter* is an indemonstrable proposition. In this interpretation, both types of hypotheses involve an assignment of truth-value. (2) A hypothesis *simpliciter* is true in itself while a relative hypothesis is true in relation to the learner. This interpretation implies that only relative hypotheses depend on an assignment of truth-value. The *Posterior Analytics* I.10 leaves unclear which of these interpretations is preferable, but other discussions throughout the *Posterior Analytics* support the first.

In the *Posterior Analytics* I.12 Aristotle says that a syllogistic question and a syllogistic premiss are the same (77a36-37). The possibility of rephrasing a syllogistic premiss as a question indicates that a syllogistic premiss can have two truth-values insofar as every question can be answered by "yes" or "no". Moreover, assigning a truth-value is the feature that distinguishes a demonstrative syllogism from a dialectical syllogism, namely an argument in which one argues for *both* parts of a contradiction indifferently; in a demonstrative syllogism, *one* of two contradictory propositions is laid down (*Prior Analytics* 24a30-b3). Accordingly, in a dialectical syllogism the premiss is posited but judgment as to its truth-value is suspended. The demonstrative premiss, by contrast, is characterized both as true and as *one* part of a contradiction; thus in demonstration the premiss is both posited and asserted as true.

Furthermore, in the *Posterior Analytics* I.10 Aristotle says:

¹¹ Aristotle's discussion of hypotheses in the *Posterior Analytics* I.10 gives further support to the relevance of "being as true" to the distinction between hypotheses and definitions. For in this chapter Aristotle refers to the truth-value assigned by the learner, in distinguishing between different types of demonstrative premisses.

What necessarily is the case because of itself and necessarily seems to be the case is not a hypothesis or a postulate. For demonstration is not addressed to external discourse – but to discourse in the soul... For one can always object to external argument, but not always to internal argument. (76b23-27)

Here Aristotle excludes unquestionable truths from being categorized as hypotheses. He stresses that every demonstrative premiss ought to be susceptible to objection, and not just to a verbal one. Being one part of a contradiction, then, means that a hypothesis is one of two possible and intelligible alternatives: demonstration involves an assertion that determines which of the two contradictory propositions is considered true. Thus, having an alternative is not the only feature of hypotheses, for to be a genuine starting point of demonstration the hypothesis should have an *intelligible* alternative

So in the *Posterior Analytics* the term hypothesis bears its original meaning: “positing as a preliminary”. Hypothesizing is an act in which one of two alternative propositions is laid down. This feature differentiates hypotheses from the other two types of first principles – definitions and axioms, which have no intelligible alternative; once they have been grasped the demonstrator is compelled to assent to them. Hypotheses require a choice between two alternatives. This feature of hypotheses finds expression in Aristotle’s characterization of a non-demonstrable premiss as “a *judgment* (*hupolêpsis*) concerning an immediate proposition” (88b37).

Viewing Aristotle’s hypotheses as involving a choice between two alternatives, Lezsl concludes that the necessity pertaining to the premisses of demonstration, i.e., to the hypotheses is a relative necessity.¹² Demonstrative knowledge is conditioned knowledge since it is based upon premisses whose truth is not self-evident. As a result, Lezsl considers the necessity of demonstration a hypothetical necessity; the demonstrative conclusion is not necessary in itself but only in relation to the premisses. Accordingly, Lezsl views demonstrative knowledge as conditioned knowledge, accepted only on the basis of unproved and non-self-evident premisses.

This conclusion, however does not accord with Aristotle’s conception of knowledge. Nowhere in the *Posterior Analytics* does Aristotle modify the necessity pertaining to knowledge; nor can Lezsl’s interpretation account for Aristotle’s distinction between hypothetical knowledge and knowledge *simpliciter* (71a10, 72b15, 74a28, 74a32). Moreover, attempting to accommodate Aristotle’s theory of demonstration to modern axiomatics, Lezsl’s interpretation emphasizes Aristotle’s views as to the truth-value of

¹² Lezsl, “Mathematics, Axiomatization and Hypotheses” (1981), p. 306 ff.

the hypotheses, while neglecting their non-formal relation to the demonstrative conclusion, which is the relation of appropriateness between the premisses and the conclusion (71b23). In the *Posterior Analytics* I.9 Aristotle argues:

Since it is evident that one cannot demonstrate anything except from its own principles if what is being proved belongs to it as that thing, knowledge is not this – if a thing is proved from what is true and non-demonstrable and immediate. (75b36–40)

This passage points to the difference between Aristotle's theory of demonstration and a modern axiomatic system, in which immediacy and truth are the sole requirements imposed upon the basic propositions of a given system. Aristotle, by contrast, considers these requirements insufficient for establishing knowledge. Besides these formal criteria, Aristotle insists on another criterion – appropriateness. A demonstrative premiss should not be just true and immediate, it has to express a special relation between subject and predicate, i.e., a necessary relation or an “in itself” relation. Such a relation can be considered only by an appeal to the specific content of the premisses.

The requirement of appropriateness stems from the main characteristic of knowledge as necessary. In the *Posterior Analytics* I.4 and 6 Aristotle discusses the necessity pertaining to demonstrative premisses.¹³ According to this discussion, a necessary proposition is a proposition in which the subject and the predicate are related to each other universally (*kata pantos*) and in itself (*kath' auto*; 73b25–28).¹⁴ A predicate may belong to a subject in itself in two cases: (1) when the predicate is part of the essence (*ti esti*) of the

¹³ Some commentators regard these chapters as a discussion of definitions; see for example, Sorabji, “Definitions: Why Necessary and in What Way?” (1981). However, although the “in itself” predication has a bearing on definitions, it is not a definition. Interpreting “in itself” predications as definitions poses a problem, for none of Aristotle's examples can be understood as definitions. “Line” is not the definition of triangle, nor is “point” a definition of “line”. Moreover, in chapter I.22, where Aristotle proves that the chain of predication is finite, he appeals to the distinction between two types of “in itself” predications (84a11–17). From this passage it is clear beyond any doubt that “in itself” relations pertain to the premisses of demonstration, and therefore to hypotheses.

¹⁴ In stressing that the predicate of demonstrative premisses should hold both universally and in itself, Aristotle bases his theory of demonstration on a stronger notion of necessity than the one found in his other writings. In other contexts Aristotle regards propositions about eternal and unchangeable things as necessary; see, for example *Metaphysics* 1015b13–15 and the detailed discussion of necessity and omnitemporality in Hintikka, *Time and Necessity* (1973). Yet in this context predication that holds universally, namely in all cases and at all times, is not demonstratively necessary, unless it also satisfies the second requirement of holding in itself. My discussion here is confined to this notion of necessity.

subject or (2) when the subject is part of the essence of the predicate. The relation between “triangle” (subject) and “line” (predicate) is an example of the first type of predication, for “line” is included in the definition of “triangle”. The relation between “line” (subject) and “straight” (predicate) is an example of the second type of predication, for “straight” is definable by the term “line”.¹⁵ Evidently, “in itself” or “in virtue of the essence” is the main feature of demonstrative necessity; within the context of Aristotle’s theory of demonstration, belonging essentially and belonging necessarily are equivalent expressions. Further, according to the *Posterior Analytics* I.6 this necessary relation pertains to the premisses of demonstration, as well as to the demonstrative conclusion (75a30). Thus, a demonstrative conclusion is necessary in two senses: being a conclusion of a valid inference, it is a necessary consequence; while as a conclusion of necessary premisses it is necessary in itself. Demonstrative knowledge is not merely hypothetical knowledge, conditioned by the non-self-evident truth of the hypotheses.¹⁶

The discussions of the existential import and the modal status of hypotheses lead to two conclusions: (1) hypotheses are propositions having a contradictory alternative (i.e., they are an apportionment of a contradiction) and (2) these propositions are necessary in themselves: in them the subject and the predicate are related *essentially*. The first conclusion determines the logical characteristic of hypotheses as propositions that can bear one of the two alternative truth-values – the true or the false. This conclusion amounts to the claim that hypotheses are subject to the principle of the excluded middle. The second conclusion relates the modal status of hypotheses as necessary propositions to an extra-logical feature, which is the essence of the subject or the predicate. In attempting to clarify the relationship between

¹⁵ These two types of necessary predications are analyzed in detail in chapter 4.

¹⁶ In light of the interrelation between necessity and essence, it might be asked how essence guarantees necessity. Does the inner relation between necessity and essence imply that the necessity of a demonstrative premiss is necessity *de re*, namely anchored in the real nature of the object? Or does it imply that essential relations are necessary *de dicto*, i.e., they determine the modal status of the proposition as a whole? The former interpretation of Aristotle’s concept of necessity as necessity *de re* is common among analytic philosophers, who claim that it stems from his essentialism. The most influential among them is Quine, *Word and Object* (1965), pp. 195–200. Among the proponents of this are Sorabji, “Definitions: Why Necessary and in What Way?” (1981); Lloyd, “Necessity and Essence in the *Posterior Analytics*” (1981). Other commentators, on the other hand, view these necessary propositions as analytic propositions: see, for example, Mansion, *Le jugement d’existence chez Aristote* (1976), pp. 62–93; Le Blond, *Logique et méthode chez Aristote* (1970), pp. 92f.; Lukasiewicz, *Aristotle’s Syllogistic from the Standpoint of Modern Logic* (1951), p. 205; Patzig, *Aristotle’s Theory of Syllogism: A Logico-Philological Study of Book A of the Prior Analytics* (1968), pp. 33–36.

these two characteristics of hypotheses, I examine the role of the principle of the excluded middle in demonstrations.

2.4 HYPOTHESES AND THE PRINCIPLE OF THE EXCLUDED MIDDLE

In the *Posterior Analytics* I.11 Aristotle argues that the principle of the excluded middle should not be assumed explicitly in demonstrative inferences. His argument to this effect is the following:

It makes no difference if one assumes that the middle term is and is not; and the same holds of the third term too. For if it is granted that that of which it is true to say man, even if not-man is also true of it – but provided only that it is true to say that a man is an animal and not not an animal – it will be true to say that Callias, even if not Callias, is nevertheless an animal and not a not animal (77a13-18).¹⁷

Any interpretation of this passage has to contend with various difficulties: does it really make no difference whether the explanatory term – the middle term – both is and is not? How can Aristotle claim that the propositions “X is a man” and the proposition “X is a not-man” are both true? How can one state that Callias is a man, even though it is true to say that he is not Callias as well? At this stage, the sole conclusion that can be drawn from this passage is that Aristotle’s conception of the principle of the excluded middle is different from the modern one. Aristotle’s discussion of the principle of contradiction in *Metaphysics* IV reveals this difference and allows a better understanding of the passage just quoted.

Metaphysics IV.4 is aimed at defending the principle of non-contradiction against relativistic objections, which might have been raised by philosophers such as Heraclitus and Protagoras. Validating the principle of non-contradiction, Aristotle avoids demonstrative arguments since they presuppose the principle of non-contradiction. Therefore, he verifies the principle of non-contradiction negatively, by examining the consequences that result from denying it. Despite the difficulties in understanding the argumentative force of this validation, Aristotle clearly bases it on the assumption that denial of the principle of non-contradiction entails the

¹⁷ Barnes’s translation is revised here. Barnes follows Ross in adding the particle *gar* at line 77a17. In my opinion, this addition weakens the argument, since the proposition “it will be true to say that Callias ... is an animal” becomes in Barnes’s translation a gloss and not a consequence.

abolishment of meaning (1006a30).¹⁸ Defending the principle of non-contradiction, Aristotle demands that his opponent's argument be meaningful both for himself and for other people. On the assumption that a meaningful discourse commits one to the principle of non-contradiction, the demand that his opponent's discourse be more than empty verbosity entails immediate endorsement of the principle of non-contradiction. On the assumption that someone's discourse is meaningful, one cannot both affirm and deny the same proposition when referring to the very same meaning. From this argument, Aristotle conceives of the principle of non-contradiction as presupposing that significant words have a definite meaning.

Further, Aristotle argues that denial of the principle of non-contradiction eradicates substance and essence (1007a20-21). He says that dismissal of the principle of non-contradiction renders all attributes accidental, since in this case there will be no such thing as being essentially man or animal (1007a21-23). Hence, the principle of non-contradiction does not presuppose only significance but specifically essential significance.¹⁹ So for Aristotle the principle of non-contradiction implies that not all attributes are accidental. This conclusion finds support in Aristotle's remark that the principle of non-contradiction applies even when a word has several meanings, provided that they are limited in number (1006a34-b2). The accounts that may be given of a term can be limited in number if and only if its essential and non-essential aspects are made distinct. For if all attributes are regarded as conveying the meaning of a given term it will have an infinite number of definitions. The term "man", for instance, may be defined as "wearing a green hat", "not wearing a green hat", "walking in the park", "reading a book", etc. In this case the possibility of forming a classification system is undermined, for if the species "man" were defined as "wearing a green hat", a particular man, such as Callias would be classified sometimes as a man and sometimes as not-man. Consequently, Aristotle's notion of the principle of non-contradiction guarantees meaning; it presupposes the basic concepts of Aristotle's philosophy, i.e., substance, essence, and the genus/species classification.²⁰

¹⁸ For a detailed analysis of Aristotle's validation of the principle of non-contradiction see Dancy, *Sense and Contradiction: A Study in Aristotle* (1975). My discussion of Aristotle's view of the principle of non-contradiction is in agreement with Dancy's conclusion that Aristotle's arguments in *Metaphysics* IV.4 are based on the following assumptions: (1) a word is significant if it has a definition and (2) a word is significant if there is an essence corresponding to it.

¹⁹ The time qualification applies to accidental attributes: a subject cannot have two contrary accidental predicates at the same time. The qualification "in the same respect" applies to essential predications.

²⁰ Lukasiewicz, ["On the Principle of Contradiction in Aristotle" (1970/1)] argues that, for Aristotle the principle of contradiction has only practical and ethical value. However, the

Having discussed Aristotle's view of the role of the principle of non-contradiction, we can turn back to Aristotle's discussion of the principle of the excluded middle in the *Posterior Analytics* I.11. This chapter opens with an implicit criticism of Plato's theory of Forms:

For there to be Forms or some one thing apart from the many is not necessary if there is to be demonstration; however, for it to be true to say that one thing holds of many is necessary. For there will be no universal if this is not the case; and if there is no universal, there will be no middle term, and so no demonstration either. There must, therefore, be some one and the same thing, non-homonymous, holding of several cases. (77a5-9)²¹

In this passage Aristotle considers the predicative relations that are presupposed by demonstrations. He distinguishes two accounts of the relation that subsists between a subject and a predicate: one is the relation subsisting between a Platonic Form and the perceptible manifold, i.e., "the one apart from the many"; the other is a non-homonymous predicative relation. The difference between these two accounts might be characterized as ontological. The Platonic expression "the one apart from the many" reflects an ontological distinction between two realms: that of being, to which the Forms belong, and that of becoming, which consists of perceptible items. From an Aristotelian perspective it may be argued that Aristotle's distinction between essential and accidental predication is an alternative to

analysis of Aristotle's arguments in favor of this law indicates that very basic Aristotelian notions depend on it. Moreover since the genus/species system is presupposed by the syllogism, the principle of non-contradiction has a logical value as well as an ontological value. I believe that Lukasiewicz's conclusion stems from modern reservations about essentialism, so Aristotle's reasons for accepting the principle of non-contradiction seem to Lukasiewicz insufficient grounds on which to base its logical value. In contrast to Lukasiewicz I follow Kant, who points out that the two basic modes of syllogism (i.e., *Barbara* and *Celarent*) are equivalent to the principle of non-contradiction. See Kant, "Die falsche Spitzfindigkeit der vier syllogistischen Figuren" (1912).

²¹ Various commentators treat this paragraph as misplaced. Ross, *Aristotle's Prior and Posterior Analytics* (1949), p. 542 suggests transferring it to chapter I.22 at lines 83a32-35; Themistius, in *Analyticorum Posteriorum Paraphrasis* (1900), p. 21 and Zabarella, in *Duos Aristotelis Libros Posteriores Analyticos Commentarii* (1966), p. 768 place it in I.8, at lines 75b24 or 75b30. Barnes, *Aristotle's Posterior Analytics* (1993), p. 144, and Mignucci, *L'Argomentazione dimostrativa in Aristotele; Commento agli Analitici secondi* (1975), p. 221, maintain that it should be placed at the end of chapter I.10. However, in my account of the relation between the principle of the excluded middle and types of predication, the discussion of non-homonymous predication *versus* the Platonic Form is relevant to the discussion of the role of axioms in demonstration. The paragraph seems to me appropriate in this context, and should not be placed elsewhere.

Plato's ontological distinction. The characterization of the Forms as eternal, unchanging, uniform, and so forth may be understood from an Aristotelian standpoint as implying that the Forms are analogous to the subject's essential attributes. Given that knowledge requires essential predications, the realm of Forms can be viewed as having an equivalent role to that of essential predications, in being an indispensable assumption for guaranteeing knowledge.

Having rejected the Platonic notion of separation, Aristotle substitutes the distinction between the "one" and the "many" by a distinction between homonymous and synonymous predications. This distinction between synonyms and homonyms is introduced in the *Categories*, in characterizing the relation between subject and predicate (1a1-12). A term is predicated synonymously of a subject, when the predicate's definition is predicated of the subject term as well. For instance, "man" is predicated synonymously of Callias, since part of the definition of "man", i.e., "animal", is predicated truly of Callias: Callias is a man as well as an animal. By contrast, the term "pale" is predicated homonymously of Callias, since the definition of "paleness", i.e., "color", cannot be truly predicated of Callias: Callias is pale but Callias is not a color. In Aristotle's analysis the Platonic ontological distinction between Forms and particulars turns into a distinction between the essential and the accidental aspects of the Aristotelian primary substance.

This discussion clarifies the relation between homonymous predications, dealt with in the opening paragraph of the *Posterior Analytics* I.11, and the principle of the excluded middle, discussed in the rest of the chapter. In the *Posterior Analytics* I.11 Aristotle maintains that it is not necessary to assume the principle of the excluded middle to make a demonstration. His argument refers to the following inference:

- (1) A man is an animal.
- (2) Callias is a man.

Therefore: (3) Callias is an animal.

Considering this inference, Aristotle argues that the effectuation of demonstrations is guaranteed only if the major term is not self-contradictory. The assumption, "A man is an animal and not a not-animal" is sufficient for concluding the proposition "Callias is an animal". But this requirement does not apply to the middle and minor terms. According to Aristotle, the fact that both propositions "Callias is a man" and "Callias is not a man" might be true does not affect the demonstration, provided that the proposition "A man is an animal and not a not-animal" is true.

Aristotle's contention that the middle term might be self-contradictory can be understood in light of the distinction made in the *Categories* between homonyms and synonyms. Viewing essence and accident as two aspects of

one and the same thing, the proposition “Callias is a man and a not-man” is not as self-contradictory as it seems. Understood from the perspective of the distinction between homonyms and synonyms, Callias is indeed a man in one respect and yet a not-man in another. Since an object has both essential and accidental attributes, one and the same predicate may belong to the same object in different respects. Consequently, it is clear why the presupposition of non-homonymous predications is sufficient for carrying out demonstrations. The major demonstrative premiss “A man is an animal” determines the meaning of the term “man”. Given that this premiss is laid down, the accidental attributes of “man” are excluded from consideration: The demonstration is based only on the essential attributes of the term. As a result, the possibility of forming the contradictory minor premiss “Callias is not a man” does not affect the demonstration. Hence, Aristotle’s claim that the principle of the excluded middle need not be explicitly assumed in demonstration is equivalent to the claim that accidental aspects should not be taken into consideration.

Further, the principle of the excluded middle applies to non-homonymous predications alone. In the *Posterior Analytics* I.11 Aristotle applies it to the premiss that relates the major term to the middle term, but does not apply it to the premiss relating the minor term to the middle term. The principle of the excluded middle applies to the relation between a species and a genus, which is an essential relation. It does not necessarily apply to the relation between the particular and its species, since a certain aspect of the particular, i.e., the accidental aspect, is not included in the species. In Aristotle’s view the principle of the excluded middle does not apply universally. Subordination to the principle of the excluded middle is a mark of essential predication: only a proposition that states an essential relation between subject and predicate can be affirmed or denied.²² Having a truth-value or in Aristotle’s terms being “one part of a contradiction”, is tantamount to being an “in-itself” predication.

²² Husik, [“Aristotle on the Law of Contradiction” (1906)] argues on the basis of this passage that syllogism is based solely on the *dictum de omni et nullo*, whereas the principle of the excluded middle has no role in syllogism. However, the *dictum* states that whatever belongs to a whole belongs as well to the parts included in it. This principle is equivalent, though not identical, to the principle *nota notae nota rei ipsius*, namely the predicate of the predicate is the predicate of the subject. As we have seen, these principles apply only to non-homonymous predication, while one of the marks of non-homonymous predication is its subordination to the principle of the excluded middle. Therefore, it is virtually meaningless to interpret Aristotle as claiming that syllogism is based on the *dictum* but not on the principle of the excluded middle. Moreover, this interpretation indicates that the principle of the excluded middle is, according to Aristotle, not a criterion of truth but of meaning.

2.5 DEMONSTRATIVE NECESSITY

Having established the relation between the principle of the excluded middle and essential predication, the question of the modal status of hypotheses can be reconsidered. Essential predications are subject to the principle of the excluded middle: they are affirmed or denied. According to this contention, a proposition that contradicts an “in itself” predication is simply false. From a modern point of view this contention seems to be inconsistent with the modal status of hypotheses. Given that an essential predication is necessary, its contradictory proposition should be not just false but impossible. In this case hypotheses are analytic propositions that have no intelligible alternative. Yet interpreting Aristotle’s notion of necessity as analytic does not accord with the characterization of hypotheses as “one part of a contradiction”.

Arguing that “in itself” predications are not analytic, Sorabji claims that analytic necessity can be known *a-priori*, while revealing an essence requires further experience.²³ *A-priori* knowledge, however, is not the main feature of an analytic proposition; it is a side effect of another feature of analytic propositions – that their negation is self-contradictory. An analytic proposition can be known *a-priori* since it has no intelligible alternative: it is asserted *a-priori* as true, since it is impossible to regard it as false without a contradiction. Examining Aristotle’s concept of necessity from this perspective, I argue that Aristotle’s necessity is not analytic since negating an “in itself” proposition does not entail self-contradiction. The following examination of the relations between definitions (i.e., essences) and hypotheses in the context of demonstrative knowledge substantiates this conclusion.

In *Metaphysics* VI.1 Aristotle distinguishes between demonstrative knowledge and the study of being *qua* being:

But all these sciences mark off some particular being – some genus, and inquire into this, but not into being simply nor *qua* being, nor do they offer any discussion of the essence of the things of which they treat; but starting from the essence – some making it plain to the sense, others assuming it as a hypothesis – they then demonstrate, more or less cogently, the essential attributes of the genus with which they deal. (1025b7-13)

According to this passage, demonstrative knowledge differs from the study of being *qua* being in two respects: (1) it does not deal with the substance, i.e., being as being, and (2) it does not investigate the essence. The second

²³ Sorabji, “Definitions: Why Necessary and in What Way?” (1981), pp. 220-225.

feature of demonstrative knowledge implies a distinction between the essence and the definition that specifies the essence. The essence and the definition are the first principles of demonstrative knowledge, yet demonstrative knowledge is not based on a preliminary inquiry into the essence. Rather, a definition, as Aristotle maintains in the *Posterior Analytics* I.2, is a thesis; it is an assumption that does not arise from an attempt to answer the question *ti esti*. Hence, the definition, which serves as a principle of demonstration, is not necessarily an accurate articulation of the real essence. Being a thesis and not a result of an inquiry that reveals the essence, the definition reveals the demonstrator's assumption as to what the real essence might be.

Bearing in mind that within the domain of demonstrative knowledge, definitions are theses, we can answer the question of the modal status of hypotheses. In the *Posterior Analytics* I.12 Aristotle distinguishes between non-geometrical questions, which concern the first principles of geometry, and geometrical questions, which refer to the proven propositions. Regarding questions of the first type, Aristotle says that they do not refute the geometer except incidentally. The qualification "except incidentally" indicates that although non-geometrical objections do not refute the geometer as a geometer, the negation of the principles of geometry is in a certain sense not self-contradictory. An incidental objection challenges the geometer's basic presuppositions and yet it is meaningful. This contention can be understood in light of the distinction between the real essence and the posited definition. Non-geometrical questions stem from different standpoints with regard to geometrical objects. These different standpoints might be characterized as arising from different definitions of the basic geometrical terms, i.e., they stem from different ways of distinguishing the essential aspect of the object from its accidental aspect. The geometer's opponent considers essential the very same aspects that the geometer considers accidental. Therefore, the opponent's objection to the first principles of geometry is incidentally meaningful, i.e., it is meaningful from the standpoint of *the opponent's* posited definitions.

This conclusion finds support in Aristotle's discussion of knowledge and opinion in the *Posterior Analytics* I.33. Accounting for the distinction between knowledge and opinion, Aristotle first states that knowledge concerns the necessary and opinion concerns the contingent (88b30-33). Yet Aristotle does not simply retain a correspondence between cognitive states and modal status. Throughout that chapter Aristotle attempts to explain how it is possible to have both opinion and knowledge about one and the same object. Drawing an analogy between opinion and knowledge on the one hand and truth and falsity on the other, Aristotle claims that despite the falsity of the proposition one can *in a sense* opine that the diagonal is commensurate

with the side of the square. Aristotle explains this claim by distinguishing the diagonal from the account that specifies what a diagonal is (*to ti en einai kata ton logon*) (89a25-32). He argues that one can have the opinion that “the diagonal is commensurate with the side of the square” since this opinion is formed on the basis of a different definition of the term “diagonal” from the one presupposed in the contrary proposition, namely “the diagonal is incommensurate with the side of the square”. Hence, Aristotle’s distinction between the diagonal itself and the definition of a diagonal enables him to construe a necessary proposition as having a false but still intelligible alternative.

Aristotle’s account of the difference between knowledge and opinion is based on the same distinction:

For the one is of animal in such a way that it cannot not be an animal, and the other in such way that it can be – e.g., if the one is of just what is a man, and the other of man but not of just what is man. (89a33-36)

Thus, the definition that serves as a first principle of a demonstrative knowledge does not necessarily convey the real essence of the defined object. The posited definition is based on an assumption that determines the essential aspect of an object. Since this definition is not derived from a prior inquiry into the real essence, an object may be defined in more than one way, according to the definer’s discrimination between the essential and accidental aspects. Hypotheses, which are “in itself” predications, are formed on the basis of the posited definition. Therefore, a hypothesis is a necessary proposition in the sense that it states an essential relation between the subject and the predicate. Nevertheless, it has an intelligible alternative, since its contradictory proposition is formed on the basis of different posited definitions. This contradictory proposition is false but not meaningless. It follows that the necessity pertaining to an “in itself” predication is not analytic, since essential predications have an intelligible alternative. Aristotle’s discussion of the distinction between knowledge and opinion indicates that if the proposition “A man is an animal” is an “in itself” predication, its negation “A man is not an animal” is not self-contradictory. These two propositions refer to the same *term*, which is conceived of from different perspectives; in the former “animal” is non-homonymously predicated of man, while in the latter it is homonymously predicated.

2.6 HYPOTHETICAL KNOWLEDGE VERSUS KNOWLEDGE SIMPLICITER

At the beginning of this chapter I argued that Aristotle's distinction between the two types of first principles, i.e., the intellect (*nous*) and the immediate proposition, is indispensable for avoiding skepticism. The foregoing analysis of the nature of hypotheses and their relation to essence and definition facilitates an elaboration of this claim. In the *Posterior Analytics* I.9 Aristotle argues that knowledge should not be identified with a proof, which proceeds from immediate, true, and non-demonstrable principles. Aristotle considers truth and immediacy, which are formal criteria, insufficient conditions for attaining knowledge. Demonstrations that are based solely upon true and immediate propositions "prove in virtue of a common feature which will also belong to something else" (75b41-42). A common feature that may belong to different subjects is an accidental predicate of each subject; such a predicate, although belonging to a certain subject, does not belong to it as such (i.e., as a member of a certain genus), since it holds also for subjects that belong to other genera. The distinction between essential and accidental predications is thus indispensable for distinguishing demonstrative premisses from merely immediate true propositions. An "in itself" predication is based on the essence, which is grasped by the intellect; therefore if a demonstration is based on intellect as well as upon immediate premisses, the distinction between demonstrative knowledge and true opinion is guaranteed. In the following I show how the distinction between essential and accidental predications enables Aristotle to establish a theory of knowledge that challenges the sophist's skepticism.

In the *Posterior Analytics* I.6 Aristotle introduces his notion of the "in itself" predication as an alternative to the sophist's skeptical position:

From this it is clear too that those people are silly who think they get their principles correctly if the proposition is reputable and true (e.g. the sophists who assume that to know is to have knowledge). (74b21-24)

The distinction between "to know" (*epistasthai*) and "to have knowledge" (*to epistêmên echein*) becomes clearer through a comparison of Aristotle's position with Plato's *Theaetetus*. In the first part of the *Theaetetus*, Plato introduces Protagoras' view that whatever one conceives of can be considered knowledge. For Plato the main shortcoming of Protagoras' view is its inability to explain the distinctions between wrong and right, good and bad, and truth and falsity. Generally, according to Plato, Protagoras fails to define knowledge, since his definition excludes the possibility of error.

Aristotle's discussion of error, in the *Posterior Analytics* I.5, can be understood as an alternative to this skeptical view.

In the *Posterior Analytics* I.5 error (*hamartia*) is defined by means of Aristotle's notion of necessity as "that which is being proved not belonging primitively and universally to the subject" (74a4-5). Under this general characterization Aristotle presents the following three examples of error: (1) proving that the sides of two right angles do not meet because the angles are right. The error, in this case, lies in supposing that the parallelism between the sides is derived from their being right angles, instead from their being equal angles; (2) proving that having two right angles belongs to isosceles triangles as isosceles triangles, and not as triangles; (3) proving that proportion alternates, i.e., that if $(a:b :: c:d)$ then $(a:c :: b:d)$, separately for numbers, lines, and solids, instead of proving it for any quantity whatsoever.

An analysis of these examples indicates that error is not distinguished from knowledge solely on formal grounds. An erroneous demonstration is characterized neither as an invalid inference nor as a false conclusion. The conclusions of the erroneous demonstrations cannot be regarded as simply false, for the proposition "The sides of two right angles do not meet" is true, and so are the propositions, "The sum of the angles of an isosceles triangle is equal to the sum of two right angles" and "If the proportion $a:b :: c:d$ holds for four lines then the proportion $a:c :: b:d$ holds for these lines as well". The error involved in demonstrating these propositions results instead from an incorrect relation between the premisses and the conclusion. Yet the erroneous relation between the premisses and the conclusion does not render the demonstration invalid.

From a formal point of view an erroneous demonstration is a valid inference; it is erroneous due to the non-formal relations that subsist between its premisses and the conclusion. More specifically, the distinction between error and knowledge is analyzed in terms of Aristotle's distinction between essential and accidental predications. A demonstration is regarded as erroneous if it fails to prove an "in itself" predication. Hence, since he distinguishes between essential and accidental predication, Aristotle is able to mark the general characteristics that distinguish knowledge from error. Knowledge differs from error neither in its truth-value nor in some external feature, such as the relation between the knower and the known. Knowledge is characterized as a special type of predication, in which subject and predicate are related to each other in a conceptual relation. According to Aristotle, predication has two facets: the truth-value of the predication and the relation between the subject and the predicate. A demonstration leading to a true conclusion that nevertheless fails to establish conceptual relations is an erroneous demonstration.

The distinction between essential and accidental predications underlies Aristotle's distinction between knowledge *simpliciter* and hypothetical knowledge. Defining knowledge *simpliciter* in the *Posterior Analytics* I.2, Aristotle enumerates the various requirements that demonstrative premisses ought to fulfill. These requirements, i.e., truth, primitiveness, immediacy, priority, familiarity, and causality, are conceived of as guaranteeing the appropriateness (*oikeios*) of the premisses to the conclusion (71b21-23). Appropriateness then is the main sign of demonstration; it distinguishes it from a merely valid inference. The characterization of the relation between the demonstrative premisses and the demonstrative conclusion in terms of *appropriateness* reveals Aristotle's non-formal conception of proof and knowledge. The first principles not only guarantee the truth-value of the conclusion, they determine the conceptual relations between the premisses and the conclusion. The conceptual relations that distinguish knowledge *simpliciter* from hypothetical knowledge cannot be determined on formal grounds alone, for one has to refer to the content of the proof in determining whether the relation between the premisses and the conclusion is essential.

Having construed demonstration as based on conceptual relations between the premisses and the conclusion, the role of the two types of first principles, i.e., the intellect and the immediate premiss, can be determined. In a modern axiomatic system, definitions determine the meaning of the basic elements to which the system refers. In such a system the definitions have no bearing on the logical relations between the premisses and the conclusion. Aristotle's definitions, by contrast, do not merely specify the meanings of the terms, they also determine the relations between the premisses and the conclusion. Demonstration requires conceptual relations between the premisses and the conclusion; thus definitions determine the relation of appropriateness that ought to subsist between the premisses and the conclusion. Since an inference is classified as demonstrative due to the conceptual relation between the premisses and the conclusion, the prior understanding of the essence determines whether a given inference is demonstrative. Elaborating on the relations subsisting in demonstration, I discuss Aristotle's theory of syllogism in the next chapter. This discussion, which focuses on Aristotle's *Prior Analytics*, is aimed at establishing the theoretical relation between Aristotle's logic and his theory of proof.

Chapter Three

The Logic of Demonstration

3.1 THE MODERN EXEGESIS OF THE THEORY OF SYLLOGISM

While setting out the program of the *Prior Analytics*, Aristotle states that this treatise aims to inquire into demonstration and demonstrative knowledge (24a1-2). Concluding the *Posterior Analytics*, Aristotle recapitulates both *Analytics*: “It is clear now what syllogism and demonstration are and how they came about” (99b15-17). Aristotle seems to consider the theory of syllogism and the theory of demonstration a unitary whole, the former serving as an indispensable stage for developing the latter. Throughout the *Posterior Analytics* syllogistic inferences exemplify and elucidate the various features of demonstrative knowledge. For instance, in the second book of the *Posterior Analytics* the attainment of knowledge of the reason why is characterized as a search for a middle term; in the *Posterior Analytics* I.14 the first syllogistic figure is said to be the most suitable for attaining knowledge (*epistêmonikon malista*); and Aristotle’s distinction between knowledge of the fact and knowledge of the reason why is exemplified by syllogistic inferences. Generally, syllogistic inferences appear to be integral to the theory of demonstrative knowledge.

Nevertheless, commentators regard Aristotle’s contention that every proof should be formulated as a syllogistic inference, as one of the major problems of interpreting his theory of knowledge.¹ Viewing the *Posterior Analytics* as a treatise concerning science, the rare occurrences of syllogistic inferences, either in Aristotle’s own scientific works or in his contemporaries’ scientific works raises the question of whether Aristotle’s theory of knowledge does indeed presuppose the theory of syllogism. Since the approach taken to this problem depends on an interpretation of Aristotle’s theory of syllogism, as well as on an interpretation of his theory of knowledge, an analysis of Aristotle’s logic precedes my discussions of the

¹ Barnes, “Aristotle’s Theory of Demonstration” (1969); Lloyd, *Aristotelian Explorations* (1996), p. 7.

theoretical implications of the theory of syllogism for the principles of knowledge, and the role of syllogistic reasoning in attaining knowledge.

In the first chapter of the *Prior Analytics* syllogism is defined as “a *logos*, in which certain things being posited, something other than what is posited follows of necessity from their being so” (24b18-20). This definition of syllogism may apply to almost every valid inference that has more than one premiss. Nevertheless, the detailed exposition of the theory of syllogism in Chapters 4-6 of the *Prior Analytics*, suggests a much narrower view of deduction. Syllogism is an inference from two predicative propositions, which relates the two terms of the conclusion to a third term, called the middle term. The subject and predicate of the conclusion are called, respectively, the minor term and the major term. Aristotle divides syllogistic inferences into three figures, which differ by the position of the middle term: in the first figure, the middle term is the subject of the first premiss and the predicate of the second premiss; in the second figure the middle term is the predicate of both premisses; and in the third figure it is the subject of both premisses. Within these figures Aristotle accepts 14 syllogistic inferences, traditionally called the syllogistic moods. Syllogistic inferences are divided into perfect syllogisms, i.e., the four syllogistic moods of the first figure, and imperfect syllogisms, i.e., the other ten syllogistic moods of the second and third figures.

The development of modern logic at the beginning of the twentieth century motivated modern commentators to abandon the ontological attitude to the theory of syllogism and to treat it as a formal system of logic.² This task was first undertaken by Lukasiewicz, who attempted to interpret Aristotle’s logic from the standpoint of modern logic. Tracing the differences between Aristotle’s logic and traditional medieval logic, Lukasiewicz construes Aristotle’s logic as an axiomatic systematization of the logical relations of inclusion, partial inclusion, disclusion, and partial disclusion. These four relations correspond to the four forms of predication, i.e., universal affirmative (all A is B), particular affirmative (some A are B), universal negative (no A is B), and particular negative (some A are not B). In this interpretation, the syllogistic moods are not rules of inference but a set of propositions, in which the first figure’s moods are axioms and the moods of the second and third figures are derived theorems. In dismissing the deductive interpretation of the theory of syllogism, Lukasiewicz argues

² For the ontological view of the theory of syllogism, see: Maier, *Die Syllogistik des Aristoteles* (1969). In certain cases my interpretation of Aristotle’s logic accords with Maier’s. Yet unlike Maier, I base my interpretation on a logical analysis rather than on metaphysical or psychological considerations. Further, as he ignores formal modern logic, Maier’s interpretation does not offer a comparison between Aristotle’s logic and modern logic.

that syllogisms are conditional propositions of the form “if [(A is B) and (B is C)] then (A is C)”. As a result, Lukasiewicz supposes a propositional logic, unknown to Aristotle, which he considers to be the underlying logic of the axiomatic system.³

The axiomatic interpretation of the *Prior Analytics* has been criticized by Smiley and Corcoran.⁴ Accounting for Aristotle’s contention that a proof is a type of syllogism, they maintain the traditional interpretation that a syllogism is a form of a deductive argument and not a conditional proposition. Stressing the argumentative role of syllogisms, Corcoran distinguishes valid arguments and sound arguments in interpreting Aristotle’s distinction between perfect and imperfect syllogisms. A valid argument consists of a set of propositions, called “premisses”, which is related to a single proposition, called the “conclusion”. Given that the conclusion follows from the premisses, the argument is valid yet it does not prove anything. In a proof, Corcoran claims, the validity of the argument should be manifested by the addition of propositions that make the chain of reasoning explicit. In this view, then, a deductive argument is not exhausted by the semantic relations (i.e., the truth-values) that subsist between the antecedent and consequent of an implication, since it has an argumentative structure as well. Accordingly, Corcoran interprets the theory of syllogism as a theory of perfecting syllogisms: its objective is to turn valid inferences into proofs by casting them into an argumentative structure. In the syllogistic system the first figure syllogisms, the rules of conversion, and the indirect syllogisms (*reductio ad impossibile*) all serve as rules of inference. The task of perfecting syllogisms is effected by the application of these rules to the second and third syllogistic figures.

Unlike Lukasiewicz’s axiomatic interpretation, the deductive interpretation of the *Prior Analytics* attempts to determine the role of syllogisms in Aristotle’s theory of demonstration. According to this interpretation, Aristotle’s theory of syllogism is intended to specify the deductive steps permitted within the system. Having an articulated specification of these deductive steps, one is able to determine whether a certain proposition follows from a set of premisses: the theory of syllogism is developed as a means of determining the validity and soundness of demonstrations. The deductive interpretation of the theory of syllogism thus supports the common interpretation of Aristotle’s theory of demonstration, according to which the *Posterior Analytics* presents a theory of an axiomatic

³ Lukasiewicz, *Aristotle’s Syllogistic from the Standpoint of Modern Logic* (1951). Lukasiewicz’s view has been elaborated in: Patzig, *Aristotle’s Theory of Syllogism: A Logico-Philological Study of Book A of the Prior Analytics* (1968).

⁴ Smiley, “What is a Syllogism?” (1973); Corcoran, “Aristotle’s Natural Deductive System” (1974).

science.⁵ In this interpretation, the role of syllogistic reasoning is restricted to imparting the truth-value of the premisses to the conclusion. Syllogistic reasoning is aimed at yielding conviction and certainty.

Nevertheless, both the axiomatic interpretation of the theory of syllogism and the deductive interpretation fail to account for the role of syllogism in Aristotle's theory of knowledge. The axiomatic interpretation of the theory of syllogism renders meaningless the question of the role of the syllogism in a theory of knowledge. Since in this view syllogisms are propositions, the theory of syllogism lacks any role in Aristotle's theory of knowledge; it rather becomes a certain axiomatic branch of knowledge that the theory of knowledge describes. By contrast, the deductive interpretation does attempt to determine the role of syllogism in the theory of demonstration. But the deductive interpretation fails to explain why Aristotle confines the mode of reasoning to syllogistic reasoning alone. Since determining the validity of inferences is an objective that can be attained by other forms of reasoning too, Aristotle could have employed a form of reasoning that is more appropriate for carrying out scientific proofs. Thus, the deductive interpretation does not explain the role of syllogism as a *special* type of deduction in Aristotle's theory of demonstration. By focusing on the unique features that set syllogistic reasoning apart from other types of reasoning, I attempt to answer the question, left open by the deductive interpretation.

3.2 SYLLOGISM AND HYPOTHETICAL DEDUCTION

According to Aristotle, the *Prior Analytics* aims to accomplish two tasks: (1) to formulate the syllogistic figures and to reduce the imperfect syllogistic moods to the moods of the first figure; (2) to resolve all types of deductions into the three syllogistic figures (46b40-47a5). In setting these two tasks Aristotle treats syllogistic reasoning as equivalent to deduction: all deductions are syllogisms, in the sense that they can be formulated or reformulated in syllogistic form (41b1-3, 44b7-8). Aristotle insists this, although it leads him to modify the formal restrictions that he set out in the *Prior Analytics* I. 4-6. The most conspicuous expression of this attitude is found in the *Prior Analytics* I.37, where Aristotle claims that the predicative relation "A belongs to B" should be understood in as many ways as the categories of being suggest. Accordingly, syllogisms are not confined to propositions having a predicative form; for instance, by applying the expression "A belongs to B" to the category of quantity, syllogistic

⁵ This view has been articulated by Scholz, "The Ancient Axiomatic Theory" (1975) and Lezsl, "Mathematics, Axiomatization and Hypotheses" (1981).

reasoning can, in his view, accommodate relational propositions such as “A equals B”. This contention casts doubt on the applicability of Aristotle’s conversion rules to such propositions. Equality, unlike predication, is a symmetric relation. Regardless of quantification, the proposition “A equals B” is equivalent to the proposition “B equals A”, whereas the predicative proposition “A is B” does not necessarily entail the proposition “B is A”.

However, this lenient attitude to formalization is qualified with regard to hypothetical deductions and *reductio ad impossibile*. Considering the following hypothetical inference: “if [(man exists) then (animal exists) and if (animal exists) then (substance exists)], then if [(man exists) then (substance exists)]”, Aristotle says:

We are deceived in such cases because something necessary results from what is assumed ... But that which is necessary is wider than syllogism; for every syllogism is necessary, but not everything that follows necessarily is a syllogism. (47a31-35)

So despite the identification of syllogism and deduction, Aristotle conceives of syllogistic reasoning as a sub-class in the wider class of deductions or necessary entailments. An analysis of the difference between syllogistic reasoning and hypothetical deductions uncovers the presuppositions that underlie Aristotle’s notion of deduction.

In the *Prior Analytics* I.44, Aristotle discusses the impossibility of reducing hypothetical deductions and *reductio ad impossibile* to one of the syllogistic figures. Commentators have interpreted this discussion as an attempt to accommodate deductions from conditional propositions within the syllogistic form.⁶ According to this interpretation, Aristotle fails to accomplish this task, since syllogistic reasoning is inadequate for the formalization of such inferences. This interpretation however does not fall into line with Aristotle’s explicit distinction between necessary entailment and syllogism. In the passage from the *Prior Analytics* I.32 quoted above, Aristotle treats hypothetical deductions as radically different from syllogistic reasoning. This approach is also adopted in the *Prior Analytics* I.44, where Aristotle stresses that hypothetical deductions should not be reduced to syllogistic deductions (50a16-17). In light of these contentions it seems implausible to interpret Aristotle as attempting to cast conditional inferences in a syllogistic form.

⁶ Kneale and Kneale, *The Development of Logic* (1962), pp. 98ff; Mueller, “Greek Mathematics and Greek Logic” (1974), pp. 35-70; Patzig, *Aristotle’s Theory of Syllogism: A Logico-Philological Study of Book A of the Prior Analytics* (1968), p. 149; Lukasiewicz, *Aristotle’s Syllogistic from the Standpoint of Modern Logic* (1951), p. 49.

Moreover, in discussing hypothetical deductions and *reductio ad impossibile*, Aristotle is by no means concerned with questions of formalization. In accounting for the difference between hypothetical deductions and syllogisms, Aristotle says that the conclusion of a hypothetical deduction is not *proved* through deduction (*ou gar dia sullogismou dedeigmenoi*) but is *asserted* through agreement (*dia sunthêkês hómologêmenoi*) (50a17-19). Aristotle regards the distinction between hypothetical deductions and syllogisms as resting on the cognitive worth of the conclusion. Comparing these two types of inference, he analyzes the distinction between them in terms of a distinction between a proof and an assertion. In a hypothetical deduction, he claims, the truth-value of the conclusion is not derived from the premisses but from the prover's agreement, while in a syllogism the truth-value of the conclusion is derived from the premisses.

Aristotle's discussion of *reductio ad impossibile* reinforces the conclusion that he does not view the distinction between hypothetical deduction and syllogism as a distinction between different logical forms. Aristotle classifies *reductio ad impossibile* as a hypothetical deduction, although it has a syllogistic form. The following inference: "All A is B, and all B is C, therefore all A is C", would be a *reductio ad impossibile* if one of its premisses contradicted the desired conclusion. For instance, in proving the proposition "some A are not B", one assumes its contradictory premiss, namely "all A is B", and establishes the desired conclusion (i.e., "some A are not B"), if the conclusion, that is derived from the contradictory premiss (i.e., "all A is B") is an evident falsehood. Although this type of argument has a syllogistic form, Aristotle maintains that only one part of it can be reduced to a syllogistic inference. According to Aristotle, the second stage of the *reductio ad impossibile* procedure, i.e., the recognition of the falsehood of the conclusion, cannot be presented syllogistically (50a29-38). In the second stage of a *reductio ad impossibile*, the truth-value of the conclusion is not entailed by the premisses but is assigned to it by the prover. Hence, it is not the formal structure of the argument that distinguishes hypothetical deductions and *reductio ad impossibile* from syllogistic inferences. They differ from syllogistic inferences in requiring the assigning of a certain truth-value to the conclusion. Aristotle appeals to this feature in distinguishing *reductio ad impossibile* from hypothetical deductions; in a hypothetical deduction the conclusion is asserted due to an agreement as to the truth-value of the premisses, whereas in a *reductio ad impossibile* the conclusion is denied since its falsehood is evident (50a36). Yet in both cases the truth-value of the conclusion is not determined by the premisses but is assigned to it by the prover.

From a modern point of view the distinction between a conclusion derived from the deduction and a conclusion derived from the assigning of a truth-value is vacuous. The basic constants of a modern system of logic, i.e., its logical connectives, are defined by their truth-conditions. For instance, a conjunction is defined as a relation between two propositions, which is true if and only if the two propositions are true; in all other cases the conjunction is false. The truth-valued definition of the logical connectives guarantees the validity of every deductive step. Having the truth table of the logical connectives, one can determine the validity of an argument by referring to the propositional truth-conditions. For instance, applying the definition of conjunction, the proposition “P and Q” is validly deduced from the proposition P and the proposition Q. Hence, in modern formal logic, deductions rest on the procedure of inferring the truth-value of the conclusion from the truth-value of the premisses. Aristotle’s discussion of hypothetical deductions indicates that he employs a different notion of deduction.

The exact notion of deduction employed by Aristotle in distinguishing hypothetical deductions from syllogisms can be understood in light of the *Posterior Analytics* II.5. Arguing that the Platonic method of division cannot be considered a deduction, Aristotle draws an analogy between hypothetical deduction and the Platonic method of division. An articulation of this analogy sheds light on Aristotle’s notion of deduction.

For just as in the case of deductions without middle terms if someone says that if these are the case it is necessary that this is the case, it is possible to ask why; so too this is possible in the case of divisional definitions. (91b37-39)

In the *Prior Analytics* I.31 Aristotle evaluates the method of division as a weak deduction, since it assumes what it has to prove (46a33-34).⁷ According to this discussion, when employing the Platonic method of division one deduces, for instance, the conclusion: “all men are mortal” from the premisses: (1) every animal is either mortal or immortal; and (2) every man is an animal. From Aristotle’s point of view, the fallacy, involved in the method of division can be characterized as deducing an assertory proposition, when only the disjunctive proposition “every man is either mortal or immortal” can be deduced. Just like hypothetical deductions, the method of division is based on the divider’s choice; the assertory proposition “all men are mortal” does not follow from the premisses but from the divider’s choice between the two disjunctive poles, i.e., mortal and immortal.

⁷ A similar criticism of the method of division is to be found in *Posterior Analytics* II.5 and *Metaphysics* VII.12.

Hypothetical deductions then are analogous to the method of division in requiring an arbitrary choice; in the former case the deduction requires a choice between the two truth-values, while the latter requires a choice between two contrary *differentiae*.

A further analysis of the analogy between the method of division and hypothetical deductions clarifies Aristotle's notion of syllogistic deduction. In *Metaphysics* VII.12 Aristotle contends that the unity of divisional definitions can be guaranteed by employing a principle of division called "the difference of difference" (1038a9-10). By this principle, the genus is divided according to the same property that distinguishes it from the other coordinate genera. For instance, while "animal" differs from all the other genera in being endowed with soul, the various species of animal differ from each other according to the nature of their soul; thus the genus animal is divided into the species: endowed with intellective soul, sensitive soul, vegetative soul. Aristotle's alternative to Plato's method of division avoids the arbitrariness involved in the Platonic method of division. According to Aristotle's principle, the division of a genus forms a system of classification, in which the genera and the species are related to each other in a conceptual relation: the concept "soul", which is an essential ingredient of the concepts "animal" and "man", relates these concepts intrinsically to each other.

Just as Aristotle's alternative to Plato's method of division implies a conceptual relation between genera and species, his notion of syllogistic deduction also presupposes a conceptual relation. Commenting on the *Prior Analytics* I.23, Alexander of Aphrodisias claims that hypothetical deductions are deficient. He explains that a conditional proposition, such as "if virtue is knowledge then it is teachable", has no cognitive value (*ouk esti gnôrimon*) unless one proves syllogistically that virtue is knowledge.⁸ In Alexander's view the antecedent of this conditional cannot be just asserted; it has to be established on the basis of this inference: (1) every state that cannot be either true or false is knowledge; (2) virtue cannot be either true or false; (3) therefore virtue is knowledge. Alexander's analysis makes hypothetical deductions superfluous, since if the antecedent and consequent of the conditional are indeed related, this relation can be proved syllogistically. But if this is so the conjunctive "if... then..." is redundant. On the other hand, if the terms are not related syllogistically the conjunctive "if... then..." is not redundant, but the premiss lacks a cognitive value, since from Alexander's point of view the terms are related arbitrarily.⁹

⁸ Alexander, *In Aristotelis Analyticorum Priorum Librum I Commentarium*, pp. 262-263.

⁹ Unlike Alexander, Aristotle does not dismiss hypothetical deductions. He treats them as a distinct type of inference.

Alexander's example, "if virtue is knowledge then it is teachable", which is one of the disputed matters in Plato's dialogues, indicates that the relation between the antecedent and the consequent is neither self-evident nor implied from the content of these propositions. The relation between the antecedent and the consequent is not a conceptual relation but is a result of agreement. Aristotle's emendation of the Platonic method of division involves the application of a principle of division that guarantees a conceptual relation between the genus and the species. Similarly, the hypothetical formulation of the premisses by means of the conjunction "if... then..." indicates that a conditional proposition does not state a conceptual relation between its components; regarding the proposition's content, the relation between the antecedent and consequent is arbitrary. The relation is not intrinsic but is based on an external feature, i.e., one's assertion of the proposition. By casting the conditional proposition in syllogistic form, it becomes necessary to make this arbitrary relation conceptual. This involves finding a middle term that can serve as a conceptual link between the two conditionally related propositions. In contrast to syllogistic premisses, a conditional premiss of the form "if A then B" yields a conclusion either by affirming the antecedent (*modus ponens*) or by negating the consequent (*modus tollens*). The conclusion of a hypothetical deduction is drawn by assigning a truth-value to one of the propositional components of the conditional premiss.

In light of this analysis, Aristotle's distinction in the *Prior Analytics* I.44, between a conclusion *proved* through deduction and a conclusion *asserted* through agreement, has to be construed as a distinction between a conclusion drawn from the content of the premisses and one drawn from the truth-value of the premisses. In syllogistic reasoning the understanding of the premisses yields the assertion of the conclusion, whereas in necessary entailment it follows from the judgment concerning the truth-value of the premisses. This interpretation of the distinction between hypothetical deduction and syllogism does not mean that syllogistic premisses lack a judgmental import: it is implied by the conceptual relation subsisting between the syllogistic terms.¹⁰ By contrast, conditional premisses, which are formed by the

¹⁰ According to Lear's interpretation of Aristotle's discussion of hypothetical deductions, the syllogistic premisses are unasserted propositions [Lear, *Aristotle and Logical Theory* (1980), pp. 34-53]. Like Lear, I do not trace the distinction between hypothetical deductions and syllogisms to the logical form. Yet contrary to Lear, I do not claim that the distinction among these types of deduction should be construed as a distinction between inferences from asserted premisses and from unasserted premisses. The distinction is rather between premisses, whose content does not imply their truth-value (i.e., hypothetical ones) *versus* propositions, whose truth-value is implied by their content (i.e., conceptual ones).

conjunction “if ... then”, give no indication as to the truth-value that should be assigned to their propositional components. Syllogistic premisses differ from conditional premisses in implying the truth-value of the proposition.

Aristotle’s conception of the relation between a proposition and its truth-value is reflected in his distinction between hypothetical deductions and *reductio ad impossibile*. Although these two types of deductions are carried out through the assignment of a truth-value to a proposition, Aristotle considers them different. As I have mentioned, the conclusion of a hypothetical deduction is asserted by agreement, whereas no such agreement is required in *reductio ad impossibile* since the falsity of the conclusion is evident. The truth-value’s dependence on the content of a proposition distinguishes these two types of deductions. In a *reductio ad impossibile* the truth-value is assigned to the conclusion on the basis of an understanding of the content. For instance, a *reductio ad impossibile* proof of the incommensurability of the diagonal leads to the conclusion that odd numbers are equal to even numbers. The falsity of this conclusion is evident from the mere understanding of the terms “odd number” and “even number”. Hypothetical deductions, by contrast, do not necessarily lead to a self-evident, or a self-contradictory conclusion. In such propositions the act of grasping the content and the acknowledgement of its truth are kept separate.

Syllogistic premisses then do have an assertive import. Yet syllogistic reasoning does not accommodate hypothetical deductions since it cannot accommodate an *explicit* assignation of a truth-value. A formulation of a proposition as a conditional proposition presupposes a distinction between the content and its truth-value, since the conjunction “if” indicates that the truth-value of the proposition is not determined. A conditional proposition expresses the independence of the truth-value from the content, while syllogistic reasoning presupposes a dependence of the truth-value upon the content.

Aristotle’s view of the relation between the content of a proposition and its truth-value constitutes the basic difference between his logic and modern logic. The distinction between a proposition and its truth-value is a fundamental assumption in Frege’s logic. Unlike Aristotle, Frege draws a sharp distinction between a thought and the truth-value assigned to this thought.¹¹ This distinction is employed in his *Begriffsschrift*, where Frege symbolizes an asserted proposition by a horizontal stroke, called the judgment stroke, in order to express recognition of the truth of the proposition.¹² The distinction between judgment and understanding is indispensable for establishing a formal system of logic. Formal logic

¹¹ Frege, *Logical Investigations* (1977), p. 7.

¹² Frege, *Begriffsschrift und andere Aufsätze* (1964), pp. 1-2.

involves an abstraction of any content from a certain argument. The assumption that the truth-value is independent of the content of a proposition underlies the possibility of considering a proposition solely as a truth-value bearer, while ignoring its content.¹³ When the focus is on the truth-value of a proposition, the different content expressed in various propositions becomes irrelevant. For instance, from the viewpoint of modern logic, the propositions “the moon is made of yellow cheese” and “the diagonal of a square is commensurate with its side” are identical, since both propositions are false. Frege expresses this view radically, claiming that a proposition is a proper name that denotes its truth-value.¹⁴ Since, as regards the denotation (*Bedeutung*) of a proposition, all true propositions are identical and all false propositions are identical, a formal description of the entailment relation can be provided, regardless of the propositional content. This discussion of hypothetical deductions indicates that Aristotle’s theory of syllogism does not share with modern logic the sharp distinction between judgment and understanding. In Aristotle’s view the truth-value and the content of a syllogistic premiss are not indifferent to one another, so he regards as non-syllogistic deductions that presuppose the distinction between judgment and understanding, such as hypothetical deductions and *reductio ad impossibile*. This conception has implications for the notion of logical consequence, with which the following section deals.

3.3 SYLLOGISTIC CONSEQUENCE

In modern logic, logical consequence is defined by the truth-value of the antecedent and consequent of an implication. The basic notion of logical consequence is commonly that of material implication,¹⁵ according to which a conclusion is logically entailed by the premisses if it is not the case that the premisses are true and the conclusion is false. Although Aristotle does not

¹³ In his axiomatization of geometry, Hilbert adopts a different view. According to Hilbert, geometrical terms can remain undefined, since their meaning is determined by the axioms. Hence Hilbert does not presuppose that judgment is independent from understanding, but regards the latter as entailed by the former. [For an explicit articulation of this approach see Hilbert’s correspondence with Frege in Frege, *Philosophical and Mathematical Correspondence* (1980), pp. 41-43.] Although Hilbert blurs the distinction between judgment and understanding, he employs Fregean logic. This logic presupposes that judgment is independent of the content; therefore a reduction of content to its truth-value, given that the truth-value has a theoretical priority, does accommodate Fregean logic.

¹⁴ Frege, “Function and Concept” (1970), pp. 31-32.

¹⁵ There are systems of logic that are based on different notions of implication, such as C.I. Lewis’s logic, which is based on the notion of strict implication. Yet this notion, like the notion of material implication, is based on a truth-value definition of implication.

define syllogistic consequence, his discussion of syllogisms from false premisses might lead to the conclusion that syllogistic entailment is based on the notion of material implication.¹⁶ In the *Prior Analytics* II.2 Aristotle says:

It is possible for the premisses of a syllogism to be true, or to be false, or to be the one true, the other false. The conclusion is either true or false necessarily. From true premisses it is not possible to draw a false conclusion; but a true conclusion may be drawn from false premisses... (53b4-8)

The truth-table of material implication can be easily constructed from this passage: Aristotle can be interpreted as claiming that a syllogistic inference is valid when (1) the premisses are false and the conclusion is true; (2) the premisses are false and the conclusion is false; (3) the premisses are true and the conclusion is true.

Aristotle's concluding remark, at the end of the discussion of syllogisms from false premisses, undermines this interpretation. In the *Prior Analytics* II.4, Aristotle asserts that it is impossible that the same thing can be necessitated by two contradictory states. In formal terms, Aristotle states that the propositions "if A then B" and "if not-A then B" cannot be true together. He supports this contention, called "Aristotle's thesis", by the following argument: Given the propositions (1) "if A then B" and (2) "if not-A then B", proposition (3) "if not-B then not-A" is inferred from proposition (1) by transposition, while from propositions (2) and (3) follows proposition (4) "if not-B then B". So (2) "if not-A then B" and (3) "if not-B then not-A" entail (4) "if not-B then B" by virtue of the so-called hypothetical syllogism. Since Aristotle regards proposition (4), i.e., "if not-B then B" as self-contradictory, he concludes that propositions (1) and (2) cannot be true together (57a36-57b16).

From the standpoint of modern logic proposition (4), i.e., "if not-B then B" is not self-contradictory but is equivalent to the proposition "B". Let us consider the following inferences:

- | | |
|--|---|
| (I) if A then B
if C then B
either A or C
therefore B | (II) if B then B
if not-B then B
either B or not-B
therefore B |
|--|---|

These two valid inferences have the same form, but they differ in their content. Inference (II) is a result of substituting B for A and not-B for C.

¹⁶ For this view see Patzig, *Aristotle's Theory of Syllogism: A Logico-Philological Study of Book A of the Prior Analytics* (1968), p. 200

“Aristotle’s thesis” is nothing but an abbreviation of inference (II), since being logical truisms, the first and the third premisses of inference (II) can be eliminated. Therefore this inference can be represented as follows: “if not-B then B therefore B”. This logical analysis of Aristotle’s thesis raises the question of why Aristotle considers the proposition “if not-B then B” self-contradictory.

From a modern point of view, Aristotle’s argument seems to be an examination of the relation that subsists between the truth-value of the premisses and the truth-value of the conclusion, although Aristotle does not employ the terms “true” and “false”. He uses the expressions, “A is white”, “B is large”, and their negations in denoting the truth-values of the propositions. More specifically, the expression “A is white” stands for the proposition “the premisses are true”; the expression “A is not white” stands for the proposition “the premisses are false”; the expression “B is large” stands for the proposition “the conclusion is true”; and the expression “B is not large” stands for the proposition “the conclusion is false”. The employment of concrete terms, i.e., “white” and “large”, seems more significant when this argument is compared with that used in supporting the thesis that a false conclusion cannot be drawn from true premisses. In constructing the latter argument Aristotle uses the expressions “A’s being” and “B’s being” in denoting the truth-values of the premisses and the conclusion respectively (53b12-14). Aristotle’s avoidance of explicit reference to truth-values in arguing for “Aristotle’s thesis” may indicate that the argument is not based on a truth-valued notion of entailment but on a content-dependent notion of entailment.

The truth-valued definition of logical consequence does not capture the intuitive notion of deduction since this definition implies that any false proposition entails every proposition and any true proposition is entailed by every proposition. For instance, the proposition “ $2+2=4$ ” is a logical consequence of the proposition “the moon is made of yellow cheese”, since the former is true and the latter is false. However, according to our intuitive notion of logical consequence the conclusion must have some conceptual relation to the premisses to be regarded as a consequence of them. From an intuitive point of view a conclusion concerning the material constituents of the moon cannot be drawn from an arithmetical proposition, since these two contents are irrelevant to each other. Similarly, a rejection of “Aristotle’s thesis” is counter-intuitive; the claim that the same proposition is entailed by two contradictory propositions does not follow the intuitive assumption that the same content cannot be derived from two opposing contents. Nevertheless, in endorsing “Aristotle’s thesis” Aristotle is not misled by an

intuitive notion of entailment, as Patzig argues.¹⁷ For Aristotle's detailed analysis of syllogisms from false premisses indicates that his notion of entailment is not based solely on the relations between truth-values.

Throughout chapters 2-4 of the *Prior Analytics* II Aristotle attempts to determine the conditions under which false premisses entail a true conclusion. As Offenberger points out, the very possibility of determining the truth-value of a conclusion, given that the premisses are false, reflects the difference between Aristotle's logic and modern formal logic.¹⁸ The employment of the truth-valued definition of logical consequence implies that if the premisses are false the conclusion *may* be true or false. In the employment of the truth-valued definition of logical consequence the definite truth-value of the conclusion cannot be determined in cases where the premisses are false. Aristotle, by contrast, specifies the conditions, under which a true conclusion is to be drawn from false premisses. For instance, regarding a first figure syllogism with two universal-affirmative premisses (*Barbara*), Aristotle states that if the two false premisses are contrary to the true premisses, i.e., wholly false, the conclusion will be true (53b31-35).

It is not difficult to trace the principle on which Aristotle sets the conditions for determining the conclusion's truth-value. In the inference "animal belongs to all stones, stone belongs to all men, therefore animal belongs to all men" the premisses are false and the conclusion is true. Yet although the premisses are false they manifest a correct relation, since the minor term is subordinated to the major. Regardless of the truth-value of the premisses a true conclusion is drawn whenever the minor term is a species subordinated to the major term. Similarly, in an inference of the form *Barbara*, a true conclusion is drawn when only the minor premiss is false. In that case the major term and the middle term are two coordinated species belonging to the same genus (54a29-34). It is unnecessary to survey each syllogistic mood to detect that syllogistic consequence is based on the relations of subordination and coordination between the syllogistic terms rather than on the truth-values of the premisses and the conclusion. Certainly, by application of the notion of material implication to syllogistic reasoning, the inferences that would be accepted and rejected correspond to the inferences that Aristotle accepts and rejects. However, a truth-valued notion of implication does not exhaust Aristotle's notion of syllogistic consequence, since its application does not enable us to account for the possibility of determining the truth-value of a conclusion derived from false premisses.

¹⁷ *Ibid.*, pp. 201-202.

¹⁸ Offenberger, "Bemerkungen zur Frage der Implikation in den Ersten Analytiken, II. Kap. 2-5" (1985).

The primacy of the relations of subordination and coordination in Aristotle's theory of syllogism finds expression in his definition of predication. In the first chapter of the *Prior Analytics* affirmative predication is defined as follows: "that one term should be in another as in a whole is the same as for the other to be predicated of all the first" (24b26-28). Furthermore, in formulating the first figure, Aristotle defines the middle term as follows: "I call that the middle, which both is itself in another and contains the other in itself" (25b35-36). The middle term is wider in extension than the minor term and narrower in extension than the major term.

From a formal standpoint, the extensional definition of the first figure's terms prevents that definition from being universally valid for all moods of this syllogistic figure. Since the relative extension of the terms can be indicated on formal grounds only in universal affirmative propositions, the extensional definition is applicable solely to a first figure syllogism with two universal affirmative premisses (*Barbara*). By contrast, the formal structure of a particular-affirmative proposition (some A are B), of a universal-negative proposition (no A is B), and of a particular-negative proposition (some A are not-B) gives no indication at all as to the relative extension of the terms appearing in them. Nevertheless, Aristotle applies the extensional criterion to universal-negative propositions. Universal-negative propositions are defined in terms of universal-affirmative propositions:

That one term should be in another as in a whole is the same as for the other to be predicated of all of the first. And we say that one term is predicated of all of the other, whenever nothing can be found of which the other term cannot be asserted; 'to be predicated of none' must be understood in the same way. (24b26-30)

Although the extension of the predicate term in a universal negative proposition cannot be determined on formal grounds, Aristotle maintains that universal-negative propositions presuppose relations of subsumption: such propositions express a denial of a relation between universal and particular.

Furthermore, while formulating syllogisms with one particular premiss (*Darii*, *Ferio*), Aristotle repeats, albeit with slight modification, the definition of the syllogistic terms: "I call that term the major in which the middle is contained and that term the minor which falls under the middle" (26a21-23). As Lukasiewicz points out, an application of this definition would exclude some valid inferences. For instance, the inference "all crows are birds and some animals are crows, therefore some animals are birds" is a valid inference having true premisses, in which the minor term "animal"

contains the middle term “crow” and does not fall under it.¹⁹ But although this inference is valid Aristotle’s definition of syllogistic terms excludes it from being a syllogistic inference. Again, this exclusion can be understood in light of the primacy of the relations of subordination in Aristotle’s theory of syllogism. The truth-valued definition of logical consequence is insufficient for determining syllogistic consequence. Apart from it, a relation of subordination is required for guaranteeing syllogistic consequence. Hence, Aristotle’s notion of logical entailment is narrower than the modern notion of entailment. From Aristotle’s point of view, the entailment relation is determined by the truth-values of the premisses and the conclusion, but also by the relations of subordination and coordination.

In formulating the second and third figures, Aristotle adopts a syntactic approach to the syllogistic terms. With regard to these figures, the middle term is defined by its grammatical role as subject or predicate in the premisses: in the second figure the middle term is the predicate of the two premisses (26b33-35), while in the third figure the middle term is the subject of both premisses. In light of the discrepancy between the semantic definition employed in the first figure and the syntactic definition employed in the other two, is the syntactic definition to be construed as an emendation of the semantic definition, aimed at accommodating it within a formal system of logic? The *Prior Analytics* I.41 suggests a negative answer:

We must not suppose that something absurd results through setting out (*ektithesthai*) the terms; for we do not use the being of this particular thing ... For in general, unless there is something related as whole to part, and something else related to this as part to whole, the prover does not prove from them, and so no deduction is formed. We use the process of setting out the terms like perception by the senses, in the interest of the learner – not as though it were impossible to demonstrate without them, as it is to demonstrate without the premisses of the deduction. (49b33-50a4)

The interpretation of this passage depends on the exact meaning of the verb *ektithenai* (i.e., to set out) in this context. Throughout the *Prior Analytics*, this verb is used with two meanings: either a certain method of exhibiting the validity of an imperfect syllogism (25a14017, 28a24-26) or extricating three terms from a verbal argument and assigning letters to them. Since the first meaning has no bearing on this passage, it is natural to follow Alexander of Aphrodisias and Philoponus in understanding the term *ekthesis* here as

¹⁹ Lukasiewicz, *Aristotle’s Syllogistic from the Standpoint of Modern Logic* (1951), p. 30.

referring to the procedure of assigning letters to concrete terms.²⁰ Moreover, chapters 32-45 of the *Prior Analytics* deal precisely with this procedure; there is no reason to assume that here Aristotle intends the other meaning. Ross, however, rejects this interpretation, stating that it fails to explain what absurdity results from assigning letters to concrete terms.²¹ Yet according to this passage a syllogism can be formed if, and only if, the syllogistic terms are subordinated to each other. Employing letters in formulating syllogisms does not necessarily convey the relations of subordination, which Aristotle regards as crucial for syllogistic deductions. Therefore, substituting letters with arbitrary terms involves the absurdity of treating non-syllogistic inferences as genuine syllogisms. Accordingly, Aristotle claims that formulating syllogisms by letters is merely a pedagogic device, employed only for the sake of representation. Hence, the extensional definition of the syllogistic terms seems to be more fundamental than their syntactic definition in Aristotle's treatment of syllogistic consequence.

The discrepancy between the semantic definition employed in the first figure and the syntactic definitions employed in the second and third figures should be interpreted, then, in light of the theoretical priority of the first figure. Developing the theory of syllogism, Aristotle reduces the ten syllogistic moods of the second and third figures to the four syllogistic moods of the first figure. The possibility of reducing the ten syllogistic moods to the four moods of the first figure guarantees their validity in the sense of altering the formal positional relations that subsist in them to the conceptual relations that underlie the first figure. Aristotle employs formal considerations but he does so only inasmuch as these relations can be reduced to conceptual relations.

The discussion has led to three conclusions so far: (1) syllogism is an inference based upon conceptual relations between its terms; (2) the truth-valued definition of logical consequence is not equivalent to syllogistic consequence; (3) the relations of subordination and coordination underlie syllogistic consequence. The first characteristic uncovers Aristotle's approach to logic; the theory of syllogism specifies the conditions under which certain content follows from another content. Basing an inference on conceptual relations means that the conclusion is derived from the understanding of the content stated in the premisses. The absence of the concept of validity from the *Prior Analytics* is an expression of this notion. In the detailed presentation of the syllogistic moods, Aristotle does not employ the distinction between valid and invalid inferences. He

²⁰ Alexander, *In Aristotelis Analyticorum Priorum Librum I Commentarium*. p. 379; Philoponus, *In Aristotelis Analytica Priora*, p. 352.

²¹ Ross, *Aristotle's Prior and Posterior Analytics* (1949), p. 413.

distinguishes combinations of premisses that do and do not entail a conclusion. The latter combinations of premisses, which are equivalent to invalid inferences, are not, in Aristotle's view, inferences at all.

The notion of validity stems from the modern conception of an inference, according to which an inference is a sequence of propositions, divided into premisses and a conclusion. This definition of an inference gives no indication at all as to the relation between the premisses and the conclusion. Since any set of premisses combined with a conclusion is considered an inference, inferences are divided into sequences having and lacking a logical relation between the premisses and the conclusion. Aristotle, by contrast, views an inference as an intrinsic relation between premisses and a conclusion. Therefore he distinguishes combinations of premisses that yield and do not necessarily yield certain content.

Lukasiewicz and Patzig consider Aristotle's extensional definition of the first figure's terms erroneous, since it is inadequate for formulating a formal system of logic.²² However, the centrality of the extensional definition does not entail a non-formal interpretation of Aristotle's logic. Formalization captures the logical form of an inference by ignoring the concrete content, stated in the inference. Aristotle's theory of syllogism accords with this description. It presents three general schemes of inference, whose logical relations do not rest on a reference to any specific content. Nevertheless, Aristotle's logic does not give rise to the development of a logical calculus that determines the validity of an argument solely on formal grounds. Basing syllogistic consequence on a relation of subordination rather than on truth-values is tantamount to formalizing the intrinsic conceptual relations between premisses and a conclusion. The relation between a part and a whole, i.e., between particular and universal subsists in any meaningful or non-tautological predication. Hence, the extensional definition of syllogistic terms is a result of a generalization of the conceptual relations that subsist in predication. The extensional definition is a formalization of the content of the inference. As a formalization of content, syllogistic inference is not formal in the sense that it is indifferent to the content; therefore from a modern point of view, it seems not to be formal. However, Aristotle's logic does not differ from modern logic in being non-formal but in its objective. Syllogistic reasoning is a formalization of understanding, whereas modern logic formalizes judgment. The former attempts to state in general terms how one content follows from another, while the latter specifies the

²² Lukasiewicz, *Aristotle's Syllogistic from the Standpoint of Modern Logic* (1951), p. 30; Patzig, *Aristotle's Theory of Syllogism: A Logico-Philological Study of Book A of the Prior Analytics* (1968), p. 98.

conditions under which one judgment (i.e., truth-value) follows from another.

3.4 THE COGNITIVE VALUE OF SYLLOGISTIC REASONING

Modern logic and Aristotle's logic stress different aspects of an inference. Modern logic offers a formal description of the relations between truth-values whereas Aristotle's describes the formal relations between concepts. The cognitive value of each system of logic is therefore different. Focusing on the judgmental aspect of inferences, modern logic provides a calculus for determining the validity of a given inference. Aristotle's logic, by contrast, does not presuppose the notion of validity. It might be asked whether the absence of the notion of validity affects the cognitive value of Aristotle's logic. The answer can be inferred from the constraints that he imposes on the syllogistic terms.

One of the celebrated examples of an Aristotelian syllogism is "all men are mortal, Socrates is a man, therefore Socrates is mortal". However, as Lukasiewicz has shown, this is not a genuine Aristotelian syllogism.²³ One of the features that distinguishes it from an Aristotelian syllogism is the singular term "Socrates" in it. In the *Prior Analytics* I.27 Aristotle confines the range of application of syllogistic terms to beings (*onta*), which are neither individuals nor highest genera, stating that individuals cannot be predicated of anything, while nothing can be predicated of highest genera (43a40-43).

The exclusion of singular terms from the theory of syllogism is one of the basic differences between Aristotle's logic and modern logic. The basic linguistic ingredients of Aristotle's logic are predicates (i.e., class concepts) while those of modern logic are names. As Russell points out, names and predicates symbolize their content in a different manner; the word "man" may function in a proposition either as a class concept or as a name. As a class concept the word "man" is a bearer of meaning, namely it symbolizes the concept of man, i.e., humanity. As a name, the word "man" denotes an object: a certain actual biped thing.²⁴ Grammatically a name can serve only as a subject of a proposition, while a class concept can serve both as a predicate and as a subject in different propositions.

The relation of denotation and the fixed grammatical role of a name are indispensable for employing variables. The employment of variables

²³ Lukasiewicz, *Aristotle's Syllogistic from the Standpoint of Modern Logic* (1951), pp. 1-3.

²⁴ Russell, *The Principles of Mathematics* (1992), pp. 42-47.

presupposes that the meaning of a term does not undergo modifications throughout an inference. Denotation does not undergo a modification of meaning, since the denoted object is fixed. Furthermore, employing variables in logic is borrowed from algebra, where general arithmetical propositions are formulated by substituting letters for concrete numbers. Such a procedure is possible with regard to numbers, since all numbers have certain common features, while the specific differences between them can be ignored. Similarly, since all proper names denote an object, all proper names can be reduced to the single expression – “this”. Hence all names can be regarded as having a common feature, while the specific differences between them, i.e., the different objects they denote, can be ignored.

Class concepts, by contrast, are bearers of meaning, and they undergo modification in different contexts. For instance, the concept of mass has different meanings in Newtonian mechanics and in relativity theory. Furthermore, class concepts undergo a modification of meaning when they change their grammatical role. Whenever a class concept serves as a subject in a proposition it reflects the content of its predicate. For instance, in the proposition “fidelity is a genetic property” the concept of fidelity loses its moral connotation while accumulating a biological one.

The formulation of a first figure syllogism requires that the same term, the middle term, serve as subject in one premiss and as predicate in the second premiss. Geach evaluates this requirement as the first stage in the history of the corruption of logic. Geach criticizes Aristotle, claiming that it is logically impossible for a term to shift about between the subject and predicate position without undergoing a change of meaning, as well as a change of role.²⁵ Yet since Aristotle’s logic is a logic of concepts and not of names, Geach’s evaluation can be rejected. Aristotle’s logic does not presuppose relations of denotation; it describes the relations between concepts. According to that logic the meaning of the middle term is indeed different in each of the syllogistic premisses. Let us consider, for instance, the following premisses: (1) every vine is broad-leaved; (2) every broad-leaved plant sheds its leaves. Here the middle term, “broad-leaved”, acquires content in proposition (2) that it lacks in proposition (1). Proposition (1) contains no indication as to the question of whether broad-leaved plants shed their leaves, while proposition (2) adds the notion of shedding leaves to the concept “broad-leaved plant”.

Nevertheless, Aristotle’s logic should not be construed as a means of creating concepts. Aristotle adopts an ontological attitude to class terms, with the result that these classes, i.e., the genera, species, and *differentiae*, are considered given. That is, concepts should be unveiled and not created.

²⁵ Geach, *Logic Matters* (1972), p. 48.

However, since Aristotle conceives of syllogism as a means of learning (e.g., 81a40), syllogistic reasoning is, from the learner's point of view, a means of acquiring concepts. Aristotle addresses this cognitive role of reasoning in the first chapter of *On the Soul*:

It seems not only useful for the discovery of the causes of the incidental properties of substances to be acquainted with the essence of those substances ... but also conversely, for the knowledge of the essence is largely promoted by an acquaintance with its properties ... (402b16-22)

So from the learner's viewpoint the essence is not a given, fixed concept. On the contrary, between the essence and the properties subsist reciprocal relations; an acquaintance with the essence makes possible the discovery of the properties, while discovery of the properties leads to an appropriate acquaintance with the essence. The essence is articulated and elucidated through the discovery of the properties of its substance. For instance, although the concept "broad-leafed plant" has a definite meaning, it is articulated and enriched when the property "shedding leaves" is added to it. Accordingly, syllogism is a bi-directional procedure; on the one hand it involves deducing properties from the essence, and on the other hand it enriches the acquaintance with the essence by explicating its properties. This procedure can be understood as a specification of the essence. The essence is an abstract concept, attained by a process of generalization, i.e., induction. Generalization yields an abstract concept since it eliminates the various features of the concrete entity. As it is a result of generalization, the abstract concept is less informative than the concrete entity. Deducing the properties by means of syllogistic reasoning enriches the abstract concept by specifying some of the properties that generalization has eliminated.

Furthermore, the Peripatetic/Stoic controversy over logic indirectly supports the contention that syllogistic reasoning involves an accumulation of content.²⁶ Commenting on Aristotle's definition of syllogism, Alexander of Aphrodisias says:

Aristotle was quite right to add that the conclusion must be something different from what has been posited. It is useless –

²⁶ On the Peripatetic/Stoic controversy over logic see Frede, "Stoic vs. Aristotelian Syllogism" (1974); Mueller, "Stoic and Peripatetic Logic" (1969). From a modern point of view, Stoic logic and Peripatetic logic can be regarded as complementary. Nevertheless, both schools not only restricted their interest to their own logic but even rejected each other's logic. This fact in itself indicates that this controversy does not merely concern formal issues, but stems also from different views as to the cognitive value of logic.

it destroys any syllogistic utility to infer what is agreed and supposed. No syllogistic utility is provided by the argument: if it is day, it is light; but it is day, therefore it is light. (18.13-16)²⁷

Alexander, then, rejects a *modus ponens* inference, holding that it does not add anything new to the content that has been already supposed in the premisses. Yet the conclusion of a *modus ponens* inference does involve a certain novelty; in the first premiss of the above-quoted inference, the proposition “it is light” is not asserted but depends upon the proposition “if it is day”. In the conclusion, the proposition “it is light” changes its status, from an unasserted into an asserted proposition. Yet in this inference, the content stated in the premisses remains the same; it does not undergo a modification. Following Aristotle, Alexander examines the deduction solely from the point of view of the content. His criticism is based on the presupposition that a logical inference should serve as a means of deducing new content from a given content.²⁸

Consequently, Aristotle’s theory of syllogism offers a general scheme that describes the internal relations between concepts: it approaches reasoning from the standpoint of the content stated in an inference. In formulating the theory of syllogism Aristotle does not appeal to the relations between truth-values but to relations between universals and particulars. Aristotle’s logic is one of understanding; unlike modern logic it does not attempt to capture the relations between judgments. Inasmuch as Aristotle’s logic describes the process of understanding, the syllogistic terms are not indifferent to their occurrence in a scheme of conceptual relations. As a result, the content of the syllogistic terms undergoes a modification throughout the syllogistic inference. In analyzing Aristotle’s concept of knowledge, in chapter five, I argue that it is this feature of syllogistic inferences that makes them an indispensable element in Aristotle’s theory of knowledge. Before elaborating upon this issue, I discuss in the next chapter the constraints that syllogistic reasoning imposes upon the first principles of

²⁷ In the English translation of Alexander’s commentary on the *Prior Analytics*, the inference that Alexander discredits as useless is rendered as follows: “If it is day, it is light; but it is day, therefore it is day”. [See Alexander of Aphrodisias, *On Aristotle’s Prior Analytics I.1-7*, (1991), p. 66.] Although this reading is supported by some manuscripts, I follow Wallies’s edition here.

²⁸ Sextus Empiricus’s criticism of the logic of his contemporaries shares the same presupposition. Assuming that logic has to increase knowledge, Sextus attempts to show that in fact it fails to accomplish this task.

knowledge. These implications of syllogistic reasoning for Aristotle's theory of knowledge are discussed through an examination of the adequacy of syllogistic inferences for the formulation of mathematical proofs.

Chapter Four

Syllogism and the Object of Knowledge

4.1 THE DISCREPANCY BETWEEN ARISTOTLE'S THEORY AND SCIENTIFIC PRACTICE

Commenting on the first proof in Euclid's *Elements*, Proclus says:

What is called "proof" (*apodeixis*) we shall find sometimes has the properties of a demonstration, in being able to establish what is sought by means of definitions as middle terms, and this is the perfect form of demonstration; but sometimes it attempts to prove by means of signs (*tekmêria*). (206.12-16)¹

In these few lines Proclus expresses the major problem raised by Aristotle's theory of demonstration, namely the discrepancy between syllogistic reasoning and scientific practice. Proclus regards syllogistic demonstrations, i.e., proofs that establish the conclusion by means of middle terms, as the perfect form of proof and explains the rare occurrences of syllogistic reasoning in Euclid's *Elements* through Aristotle's notion of proofs based on signs. Proclus assumes that Aristotle's *Posterior Analytics* aims to provide the theoretical basis of scientific practice. He attempts to resolve the discrepancy between Aristotle's theory of demonstrative knowledge and Euclid's mathematics by grounding the latter in another Aristotelian notion, that of proofs based on signs.

Proclus' problem reflects a general difficulty that extends beyond the relation between Aristotle's theory of demonstration and Greek mathematics. Aristotle's theory of demonstration seems inconsistent even with his own scientific treatises. More specifically, the prominence that Aristotle ascribes to syllogistic reasoning in the *Posterior Analytics* does not find expression in his physical or biological writings. Faced with this inconsistency, Aristotle's modern commentators adopt one of two attitudes: one view rejects the assumption that the *Posterior Analytics* suggests a theory of scientific

¹ The English translation is quoted from Proclus, *A Commentary on the First Book of Euclid's Elements* (1970), p. 161.

methodology. The other endorses this assumption, while attempting to minimize the discrepancy between Aristotle's theory and scientific practice.²

The commentators of the first group claim that syllogistic reasoning is not central to the theory of demonstration. This interpretation is based on a developmental hypothesis, according to which the theory of syllogism was developed prior to the theory of demonstration.³ By this interpretation syllogistic reasoning imposes no constraints on the major aspects of the theory of demonstration, such as the first principles of knowledge.⁴ The role of syllogistic inferences in Aristotle's theory of knowledge is confined to presentation and imparting the results of knowledge already attained. Syllogistic reasoning is held to be a pedagogical device, which has no bearing on scientific activity.

As noted, commentators of the second group attempt to minimize the discrepancy between Aristotle's theory and his own scientific practice. Focusing on Aristotle's biological writings, some commentators argue that certain central aspects of Aristotle's theory of demonstration, such as the notions of definition, explanation, and axiomatization, are in fact exemplified in Aristotle's biological studies.⁵ The rare occurrences of syllogistic inferences throughout the biological works led to the suggestion that the theory of syllogism provides a formal pattern in which the scientific results could be cast. But this attempt to reconcile the discrepancy fails to account for the crucial role of syllogistic reasoning in Aristotle's theory of demonstration. This interpretation, like the pedagogical interpretation, treats

² The attitude adopted by Aristotle's modern commentators is significantly different from Proclus's attitude. Whereas Proclus resolves the discrepancy between Aristotle's theory of demonstration and Euclid's *Elements* by accommodating the latter to the former, Aristotle's modern commentators attempt to interpret Aristotle's theory of knowledge, so that it will match scientific practice. The difference between the ancient and modern attitudes reflects different conceptions of the relationship between philosophy and science. Modern philosophy of science attempts to provide an adequate analysis of scientific notions and methods. Proclus, by contrast, seems to assume that philosophy provides a model, to which science has to conform.

³ Solmsen, *Die Entwicklung der aristotelischen Logik und Rhetorik* (1929), p. 269; Barnes, "Aristotle's Theory of Demonstration" (1969), p. 66; "Proof and Syllogism" (1981), pp. 33-57; Smith, "The Relationship of Aristotle's Two *Analytics*" (1982); endorsing an alternative view, Ross, *Aristotle's Prior and Posterior Analytics* (1949) offers a detailed reply to Solmsen's textual arguments (see pp. 7-22).

⁴ Barnes, "Proof and Syllogism" (1981). By contrast, an interpretation that argues for a strict theoretical relationship between the first principles of demonstration and syllogistic reasoning is presented, in Hintikka, "On the Ingredients of an Aristotelian Science" (1972).

⁵ Bolton, "Definition and Scientific Method in Aristotle's *Posterior Analytics* II.7-10" (1987); Lennox, "Divide and Explain: The *Posterior Analytics* in Practice" (1987); Gotthelf, "First Principles in Aristotle's *Parts of Animals*" (1987).

sylogistic reasoning as merely a form of presentation, indifferent to scientific activity and scientific results.

The controversy between these two interpretations concerns the relationships between Aristotle's theory of demonstration and scientific practice. Although each interpretation suggests different answers to this question, they share the view of the role of syllogism in Aristotle's theory of demonstration. Viewing syllogism as a form of presentation, employed either for pedagogical purposes or for the sake of systematization, these interpretations presuppose that the syllogistic form of reasoning is indifferent to its content. But the foregoing analysis of Aristotle's theory of syllogism indicates that syllogistic reasoning is not indifferent to the content that it formalizes. It seems to have more bearing on Aristotle's theory of demonstration than either of the above interpretations suggest.

Focusing on the theoretical relationship between Aristotle's logic and his theory of demonstration, I adopt a different attitude to the discrepancy between Aristotle's theory of demonstration and scientific practice. Instead of settling it I try to trace its origins. I regard Aristotle's theory of demonstration as a normative theory: it does not aim to accommodate scientific practice but represents Aristotle's view of what science should be. A comparative analysis of mathematical reasoning with syllogistic reasoning discloses the different theoretical presuppositions that underlie Aristotle's theory of demonstration and Greek mathematical practice. The relationship between syllogistic reasoning and Aristotle's theory of demonstration is determined in light of this comparison.

4.2 SYLLOGISM AND GREEK MATHEMATICAL REASONING

Despite the absence of historical evidence the *Posterior Analytics* and Euclid's *Elements* are generally regarded as resting on common theoretical foundations. This view is based mainly on the parallelism between the axiomatic structure of the *Elements* and Aristotle's account of the first principles. Euclid's *Elements* seems to be structured in accordance with Aristotle's characterization of a demonstrative branch of knowledge. Apart from employing the basic distinction between first principles and derived theorems, Euclid classifies the first principles under three groups: definitions, common notions, and postulates. This threefold classification seems to parallel to Aristotle's distinction of definitions, axioms, and

hypotheses.⁶ Aristotle views the axioms as common to more than one branch of knowledge, and Euclid's common notions too, such as "if equals are added to equals the wholes are equal", can be extended beyond geometry. Further, Aristotle exemplifies axioms by the proposition "if equals be taken from equals the remainders are equal", which is one of Euclid's common notions (76a42; 77a30; 1061b9). Second, Aristotle's hypotheses, which are traditionally interpreted as existence claims, seem to correspond to Euclid's postulates, which imply the existence of certain figures by assuming the possibility of their construction. Finally, the definitions in both treatises are considered a specification of the meanings of the basic terms. Besides the similarity of method, further evidence for theoretical interrelations between Aristotle's theory of demonstration and Greek mathematics is the resemblance between Aristotle's terminology and Greek mathematical terminology,⁷ as well as the many mathematical examples scattered throughout the *Posterior Analytics*.

However, in respect of the forms of reasoning the above parallelism is undermined. Not only is syllogistic reasoning absent from Euclid's *Elements*, but it also seems inadequate to the task of carrying out a mathematical proof. Such inadequacy raises the question of whether the parallelism between Aristotle's theory of demonstration and Euclid's *Elements* is confined to the axiomatic structure alone. If Aristotle's theory of demonstration is based on his theory of syllogism, that might affect his views of the role and content of the first principles. In that case Aristotle's theory of demonstration differs radically from Greek mathematical practice. The following attempt to consider this question traces the grounds for the inadequacy of syllogistic reasoning for mathematical reasoning to the difference between the theoretical presuppositions that underlie each form of reasoning. This discussion concludes that Aristotle's theory of demonstration cannot be made to accommodate Greek mathematics.

The discrepancy between syllogistic reasoning and mathematical proofs extends beyond the problem of formalization. From a formal point of view, the task of rephrasing mathematical inferences so that they comprise two predicative propositions, which relate three terms, is indeed achievable. Down the ages, Aristotle's successors have repeatedly undertaken such a

⁶ Among the proponents of this view are: Heath, *Mathematics in Aristotle* (1970), pp. 53-57; Hintikka, "Aristotelian and Geometrical Axioms" (1981) pp. 142-143; von Fritz, "Die *Archai* in der griechischen Mathematik", (1971), p. 373; Lee, "Geometrical Method and Aristotle's Account of First Principles" (1935), pp. 115-118; McKirahan, *Principles and Proofs – Aristotle's Theory of Demonstrative Science* (1992), pp. 144-149.

⁷ On this see Einarson, "On Certain Mathematical Terms in Aristotle's Logic" (1936).

task.⁸ Furthermore, Aristotle does not restrict syllogistic reasoning to simple predicative propositions of the form “A belongs to B”. In the *Prior Analytics* I.37 he maintains that the expression “A belongs to B” should not be restricted to the category of substance; it can also convey the meanings of any other category. Given Aristotle’s lenient attitude to formalization, all Euclid’s theorems could seemingly be reformulated as syllogistic premisses; mathematical relations, such as equality or congruence, could be formulated syllogistically. Yet Aristotle’s discussion of these syllogisms indicates that such relations are ambiguously expressed in the syllogistic inferences. In the *Prior Analytics* I.36 Aristotle says:

For we state this universally without qualification, that the terms ought always to be stated in the nominative cases, e.g. man, good, contraries, not in oblique cases, e.g. of man, of good, of contraries, but the propositions ought to be understood with reference to the cases of each term – either the dative, e.g. ‘equal to this’, or the genitive, e.g. ‘double of this’, or the accusative, e.g. ‘that which strikes or sees this’, or the nominative, e.g. ‘man is an animal’, or in whatever other way the word falls in the proposition. (48b39-49a5)

Here Aristotle offers different accounts for the syllogistic terms and for the syllogistic premisses. He argues that the syllogistic terms ought to be presented in the nominative case, although the syllogistic premisses express relations. So the formal structure of the syllogistic inference, which is uncovered by the extrication of the terms, does not convey the relations that the premisses connote. Giving different accounts for terms and for premisses, Aristotle detaches the formal structure of the inference from its content. Accordingly, Euclid’s mathematical theorems can be transformed into syllogistic inferences, but this will not preserve the correspondence between the form of inference and the content that it expresses. Syllogistic rephrasing of mathematical proofs might not adequately convey the content of mathematical proofs. Comparing Greek logic to Greek geometry, Mueller points out that casting a mathematical proof in syllogistic form affects the content of the mathematical proof. More specifically, Mueller argues that spatial relations between geometrical objects do not find expression in the syllogistic rephrasing.⁹

⁸ Aristotle’s Greek commentators sometimes paraphrase mathematical proofs in such a way as to impose syllogistic form on these arguments. For instance, see Alexander, in *Aristotelis Analyticorum Priorum Librum I Commentarium*, 344.13-20. Of all the attempts to put mathematical proofs into syllogistic form, the most elaborate that I know of is found in: Herlinus and Dasypodius, *Analyseis Geometricae Sex Librum Euclidis* (1566).

⁹ Mueller, “Greek Mathematics and Greek Logic” (1974), p. 52.

In accounting for the inadequacy of syllogistic reasoning to mathematical proofs, Mueller suggests that it stems from Aristotle's lack of concern with formalization.¹⁰ But as the above discussion has indicated, regarding rephrasing Aristotle does not restrict syllogistic inferences to predicative premisses. Yet he does insist that the inference's structure, which is expressed by the syllogistic terms, should retain the predicative form. Aristotle thereby appears to regard his formal theory of syllogism as underlying inferences from relational premisses too. It would seem that the reasons why syllogistic inferences are inadequate to the task of carrying out mathematical proofs could be found in the theory of syllogism rather than in the form of the syllogistic premisses. A comparison between syllogistic inferences and mathematical proofs confirms this contention.

In considering the possibility of formulating a mathematical proof syllogistically, I discuss Aristotle's formalization in the *Posterior Analytics* II.11. There Aristotle exemplifies how the various senses of the term "cause" are expressed in syllogistic demonstration. Clarifying the notion of hypothetical necessity, he presents the following geometrical example:

Why is the angle in the semicircle right? It is right if what holds? Well, let right angle be A; half of two rights B; the angle in the semicircle C. Thus B is the cause that A, right angle, belongs to C the angle in the semicircle. For this is equal to A and C to B; for it is half of two rights. So if B, half of two rights, holds, then A belongs to C (that is, the angle in the semicircle is right). (94a28-31)

From this passage the following first figure syllogism can be formulated:

- (1) A right angle belongs to all halves of two right angles;
- (2) Half of two right angles belongs to the angle in the semicircle;

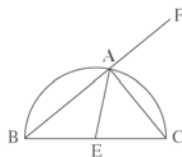
Therefore, (3) a right angle belongs to the angle in the semicircle.

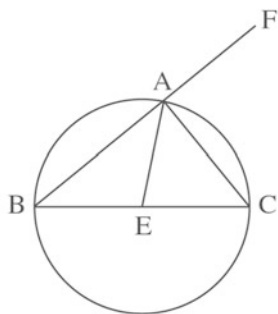
The Euclidean proof of this theorem (*Elements* III.31) has three parts, of which only the first is relevant to this comparison; therefore only this part will be considered here:¹¹

¹⁰ Ibid., p. 54.

¹¹ In accounting for this Aristotelian passage, Heath refers to the following proof which is found interpolated into the Euclidean text [Heath, *Mathematics in Aristotle* (1970), pp. 71-73]:

- (1) The exterior angle AEC of the triangle ABE is equal to the sum of the interior angles EAB, EBA (Euclid, I.32).
- (2) Since EA is equal to EB, the angles EAB and ABE are equal.





In a circle the angle in the semicircle is right.

Let ABDC be a circle, let BC be its diameter and E its center ... I say that the angle BAC in the semicircle is right.

- (1) Let AE be joined and let BA be carried through to F.
- (2) Then, since BE is equal to EA, the angle ABC is equal to the angle BAE (Euc. I.5; base angles of an isosceles triangle are equal).
- (3) Since CE is equal to EA, the angle ACE is also equal to angle CAE (Euc. I.5).
- (4) Therefore, the whole angle BAC is equal to the two angles ABC and ACB.
- (5) The angle FAC exterior to the angle ABC is also equal to the angles ABC and ACB (Euc. I.32; exterior angles are equal to the sum of the opposite interior angles).
- (6) Therefore the angle BAC is equal to the angle FAC.
- (7) Therefore each is right (Euc. def. I.10).
- (8) Therefore the angle BAC in the semicircles is right.¹²

An examination of Aristotle's proof in light of the Euclid's proof reveals conspicuous differences between the two. Aristotle's syllogistic proof can be made to correspond to steps 6-8 of the Euclidean proof: in both, the conclusion, which concerns the equality of the angle in the semicircle to a right angle, is deduced from a similar definition of a right angle, i.e., "a right

- (3) Therefore the exterior angle AEC is double the angle EAB.
- (4) For the same reason, the angle AEB, being the exterior angle of the triangle AEC, is equal to double the angle EAC.
- (5) The sum of the two angles AEB, AEC are together equal to two right angles (Euclid I.13).
- (6) Therefore the angle in the semicircle BAC is equal to half of two right angles and is therefore a right angle.

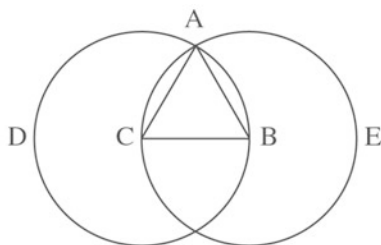
This proof may indeed correspond better to the *Posterior Analytics* II.11. Still, my analysis of the discrepancy between mathematical proofs and syllogistic inferences holds for this proof as well.

¹² The elucidations in parentheses are my interpolations.

angle is half of two right angles".¹³ Apart from this correspondence, the two proofs differ fundamentally. Compared with Euclid's proof Aristotle's syllogistic inference does not appear to be a proof at all. In Aristotle's proof it is assumed that the angle in the semicircle is equal to half of two right angles; this assumption is tantamount to the conclusion that the angle in the semicircle is right. In Euclid's proof the equality of the angle in the semicircle to a right angle is established by showing the equality between angle BAC and angle FAC. Further, a comparison between Aristotle's inference and the corresponding steps in Euclid's proof, i.e., steps 6-8, leads to the same conclusion. In Euclid's proof as well as in Aristotle's syllogistic inference the definition of a right angle is applied to the angle in the semicircle. Yet in Aristotle's inference the applicability of this definition to the angle in the semicircle is not established. By contrast, in Euclid's proof the applicability of the definition of a right angle to angle BAC is derived from the construction, accomplished in step (1). Line BF is shown to be straight, since it is produced from line AB, according to postulate I.2.

Aristotle presents his syllogistic rephrasing of this geometrical proof as an answer to the question "why is the angle in the semicircle right?" (94a28). In Aristotle's syllogistic inference the question of *whether* the angle in the semicircle is right has already been answered: he explicitly assumes at the outset that the angle in the semicircle is right. This attitude might seem to be determined by the context, in which Aristotle refers to the geometrical proof discussed above. In the *Posterior Analytics* II.11 Aristotle attempts to show how the cause is revealed in demonstrations. Therefore, in this context it might be natural to neglect the question of whether the angle in the semicircle is right, while focusing on the question of why it is right. But this cannot explain away the discrepancies between syllogistic inferences and geometrical proofs, since other syllogistic reformulations of geometrical proofs manifest the same discrepancies. In his commentary on Aristotle's *Prior Analytics*, Alexander of Aphrodisias formulates syllogistically a piece of reasoning from *Elements* I.1, which concerns the construction of an equilateral triangle.

¹³ Euclid's definition of "right angle" is as follows: "When a straight line set up on a straight line makes the adjacent angles equal to one another, each of the equal angles is right and the straight line standing on the other is called perpendicular to that on which it stands" [Heath, *The Thirteen Books of Euclid's Elements* (1956), vol. 1, p. 181].



- (1) Things equal to the same thing are also equal to each other;
- (2) Sides CA and CB of the triangle are things equal to the same thing, i.e., to side AB;

Therefore, (3) CA and CB are equal to each other (see, 344.13-20).

In Alexander's inference, as in Euclid's, the equality of sides CA and CB is inferred from the general principle "things equal to the same thing are also equal to each other". But here, as in Aristotle's formulation, the contention that sides CA and CB are in fact equal to the same thing is not proved but is assumed in premiss (2). In *Elements* I.1, by contrast, the equality of sides CA and CB to side AB is established by construction. The analyses of Aristotle and Alexander's syllogistic representations of Euclidean proofs indicate that when cast in a syllogistic form, the geometrical proofs lose their heuristic character. Syllogistic inference depends on a subsumption of a particular case under a certain general principle, while the applicability of a general principle to a particular instance is not established. By contrast, establishing the applicability of general theorems to a particular case is the core task of mathematical proofs. This discrepancy between syllogistic reasoning and mathematical reasoning cannot be settled within the constraints of syllogistic reasoning. Establishing the applicability of general theorems to particular cases requires a form of reasoning that can prove the relation of subsumption between the major and the minor premiss. As I have shown in the previous chapter, the relation of subsumption is presupposed by syllogistic reasoning, so any syllogistic inference constitutively assumes the relations of subsumption at the outset. The inadequacy of syllogistic reasoning to mathematical proofs stems from the syllogistic notion of entailment, which presupposes the relation of subsumption. As a result, syllogistic inferences cannot be made to accommodate mathematical proofs by the introduction of mathematical relations, such as equality, into the syllogistic premisses. The substitution of the predicative premisses by relation-terms does not affect the syllogistic relation of entailment, subsisting between the premisses and the

conclusion.¹⁴ Further evidence in support of this interpretation is introduced in the following section concerning the relationship between syllogistic reasoning and the premisses of demonstration.

4.3 SYLLOGISTIC LOGIC AND THE PREMISES OF DEMONSTRATION

In the *Posterior Analytics* I.4 Aristotle specifies the requirements imposed upon demonstrative immediate propositions. He introduces the notion of universal predication, which is exemplified by the relation between man and animal, i.e., the predicative relation between a genus and its species. Aristotle's characterization of universal predication accords with his definition of predication, introduced in the opening chapter of the *Prior Analytics*, where universal predication is defined in terms of the relation between a whole and a part. However, the essential predications, on which demonstrative syllogisms are based, are not identical with the relation, expressed in universal predications. Demonstrative syllogisms are based on an "in itself" predication, which according to Aristotle holds in two cases:

¹⁴ In his paper "Making sense of Aristotelian Demonstration" (1998), Henry Mendell attempts to explain Aristotle's contention that the mathematical sciences carry out their proofs syllogistically, arguing that Aristotle admits a non-syllogistic form of reasoning, which can be put into the first syllogistic figure. Mendell's argument is based on several passages, such as the above-quoted passage from the *Prior Analytics* I.36, in which Aristotle loosens the formal constraints imposed on the structure of syllogistic inferences. From these passages Mendell concludes that the predicative relation "A belongs to B" can be read flexibly so that almost any *two-place predicate* can serve as the belonging relation. In attempting to accommodate Aristotelian demonstrations to mathematical proofs, Mendell focuses on the formal aspect of syllogistic and relational inferences, while ignoring their non-formal aspect. As a result, Mendell is not sensitive to the implications of the difference between relations, viewed as two place functions and relations that are part of the predicate term. For example, although Mendell is aware that the statement " x is equal to y " can be analyzed as referring either to (1) two terms x and y , which are related by "equal", or to (2) the terms y and x 's being equal, he does not apply this distinction in interpreting Aristotle's mathematical examples. For instance, in interpreting Aristotle's syllogistic formulation of the proof of the equality of the angle in the semicircle to a right angle, Mendell claims, "here 'belongs' clearly means 'is equal to'" (p. 174). This interpretation is based on an unclear preference for the first analysis of the statement " x is equal to y " over the second. That Aristotle allows the introduction of relational terms such as "equal to" into syllogistic inferences does not imply that he considers the term "equal" a relation between two entities (or classes); in the context of Aristotle's predicative logic it can also imply that the term "equal" is a constituent of the complex term, "equal to- y " that serves as the predicated of the subject x . Mendell is aware of the plausibility of this interpretation, but he does not take it seriously into account, in addressing the applicability of Aristotle's logic to mathematics.

(1) the predicate is the definition or part of the definition of the subject; (2) the subject is part of the definition of the predicate. An analysis of these relations indicates that the formulation of these predicative relations reflects an intrinsic discrepancy between the object of knowledge and syllogistic reasoning.

Aristotle characterizes the first sense of an “in itself” predication:

One thing belongs to another in itself, both if it belongs to it in what it is – e.g. line to triangle and point to line (for their substance is from these and they belong in the account which says what they are). (73a34-37)

The difficulty arising from this characterization of an “in itself” predication is manifested in the parenthetical gloss; the relations that hold between “line” and “triangle”, or “point” and “line” are explained in two different ways: one explanation appeals to the constituents of the figure, whereas the other appeals to the constituents of the definition. These two explanations are incompatible. In *Metaphysics* III.3, Aristotle asks whether the genus or the material elements should be considered first principles. Concluding this discussion, he says:

But, again, it is not possible to describe the principles in both ways. For the formula of the substance is one; but definition by genera will be different from that which states the constituent parts of a thing. (998b11-14)

Hence, although Aristotle conceives of these two types of definitions as incompatible, in the *Posterior Analytics* I.4, he considers them mutually consistent.

The discrepancy between the views expressed in *Metaphysics* III.3 and in the *Posterior Analytics* I.4 can be explained in light of the requirements that syllogistic reasoning imposes on demonstrative premisses. Syllogistic reasoning is based on predicative relations. By contrast, the relations subsisting among geometrical objects are not predicative but spatial, such as congruence or incidence. In forming his theory of demonstration, Aristotle attempts to reduce the spatial relations that subsist among geometrical objects to the predicative relations that subsist between genera and species. Unlike their role in geometry, geometrical terms are not regarded, in the *Posterior Analytics*, as the constituents from which geometrical figures are formed; they are conceived of as genera, under which specific geometrical figures are subsumed. Generally, the intrinsic discrepancy between the objects of knowledge and the theory of substance is settled by accommodating the objects of knowledge to the theory of substance. Within the scope of Aristotle’s theory of demonstration this attitude finds expression

in the systematic attempt to reduce non-conceptual relations, such as spatial relations, to syllogistic relations. Aristotle's theory of demonstration manifests an attempt to accommodate the objects of knowledge to syllogistic reasoning, and not *vice versa*. As a result, the unique characteristics of the objects of knowledge, such as spatial relations in the case of mathematical objects, do not find expression in this form of reasoning. Syllogistic reasoning, then, seems to be the fundamental element in Aristotle's theory of demonstration.

Similarly, the intrinsic discrepancy between the objects of knowledge and syllogistic reasoning finds expression in the second sense of "in itself" predication. This sense of an "in itself" predicative relation subsists between terms like "curved" and "line", or "odd" and "number". In these cases, the subject of predication, i.e., "line" or "number", is included in the definition of the predicates, "curved" and "odd". This sense of an "in itself" predication is distinguished in *Metaphysics* VII.4 from the essence and the substance. Aristotle qualifies the contention that an "in itself" predication exhausts the essence of the substance:

But not the whole of this is the essence of a thing; not that which something is in virtue of itself in the way in which a surface is white, because being a surface is not being white. (1029b16-18)

Although "white" is not the essence of a surface, the relation between "white" and "surface" is not accidental since as a color, white always coexists with surface. Yet the persisting relations between color and surface are not conceptual relations; nothing in each concept (i.e., in the definition) implies the relation between them. Nevertheless, in the *Posterior Analytics* Aristotle conceives of such relations as essential relations. The examples that are introduced in clarifying the second sense of "in itself" predications suggest that Aristotle has essential relations in mind – odd is a species of the genus number, so number is part of its definition; similarly, curved is a species of the genus line. But shortly afterwards Aristotle introduces the proposition "the sum of the angles of a triangle is equal to two right angles", as another example of this type of "in itself" predications (73b31). Yet the relation between "two right angles" and "a triangle" cannot be construed as a genus/species relation; nor is either of these terms required for defining the other. Still, Aristotle conceives of the proposition "the sum of the angles of a triangle is equal to two right angles" as exhibiting this sense of "in itself" relations.

The discrepancy between Aristotle's definitions of "in itself" relations and the examples he gives in elucidating these relations is another expression of the discrepancy between syllogistic reasoning and the object of

knowledge. Aristotle's attitude to the objects of knowledge is dominated by his attempt to reduce non-conceptual relations to conceptual relations. This attitude can be construed as an attempt to accommodate non-conceptual relations to syllogistic reasoning. Aristotle's discussion of mathematical necessity in *Physics* II.9 shows how such a reduction is effected. He describes the proof that the sum of the angles of a triangle is equal to two right angles: "Since a straight line is what it is, it is necessary that the angles of a triangle should equal two right angles" (200a16-18). Although the relation between the sum of the angles of a triangle and two right angles is not a conceptual relation, Aristotle conceives of it as derived from the definition of a straight line.¹⁵ The rationale behind his description of this geometrical proof might be reconstructed as follows. In the assumption that a triangle is a species of the genus line, "line" is regarded as part of the essence of a triangle. As a result, the sum of the angles of a triangle is held to be derived from the essence of a triangle. In viewing this theorem as derived from the definition of a line, Aristotle both follows geometrical practice and accommodates it to his conception of substance. The geometrical proof of this theorem is based on drawing a straight line parallel to the base, or parallel to one of the triangle's sides. In both versions, proving the theorem requires drawing a straight line. While formulating the proof concerning the sum of the angles of a triangle, Aristotle conceptualizes the constructional step of drawing a straight line. He reduces the spatial relation between the triangle and the drawn straight line to a conceptual relation between the genus line and the species triangle.

In casting mathematical proofs in syllogistic form, Aristotle ascribes priority to predicative relations over others, such as spatial relations that underlie geometrical reasoning. His claim in the *Posterior Analytics* I.14 that the mathematical sciences carry out their proofs through the first figure is not an attempt to accommodate the theory of syllogism to scientific practice. On the contrary, Aristotle's treatment of non-predicative relations, such as the relation between a whole and its components, manifests a reverse attempt; i.e., to accommodate these relations to those subsisting in essential predications. The following analysis of Aristotle's conception of the objects of knowledge places his view of the priority of predicative relations over non-predicative relations within the broader context of his metaphysics. By discussing Aristotle's conception of mathematical objects as the results of abstraction, I argue that the discrepancy between Aristotle's theory of demonstration and scientific practice emerges from a tension, inherent in his

¹⁵ In *Metaphysics* V.30 Aristotle regards the relation between the sum of the angles of a triangle and two right angles as "*per se* incidental": a predicate that belongs to the subject in itself, but it is not part of the subject's substance (1025a30-34).

metaphysics, between the ontological status of the objects of knowledge and their cognitive worth. This analysis shows that the reduction of non-essential relations to essential relations, as found in the *Posterior Analytics*, is consistent with Aristotle's treatment of the objects of knowledge in general and the objects of mathematics in particular.

4.4 THE OBJECTS OF MATHEMATICS AND SYLLOGISTIC REASONING

Aristotle often refers to the objects of mathematics, with the expression *ta ex apaireseos* – the results of abstraction.¹⁶ This expression alludes both to the ontological status assigned to the mathematical objects and to the method by which mathematical knowledge is attained. To understand the ontological and epistemic characteristics of mathematical objects as the results of abstractions, it is necessary to consider Aristotle's notion of these objects in its context. His most elaborate discussion of the objects of mathematics is found in *Metaphysics* XIII. 1-3, where he deals with the following question:

We have stated what is the substance of sensible things, dealing in the treatise on physics with matter, and later with the substance which has actual existence. Now since our inquiry is whether there is or is not besides the sensible substances any which is immovable and eternal, and, if there is, what it is ... (1076a8-12)

Aristotle's preface to the discussion of mathematical objects indicates that this issue is an integral part of his exploration of substance in *Metaphysics* VII-IX. His interest in the objects of mathematics is motivated by the question of whether there are other sublunary substances besides perceptible substances. The question of the ontological status of the objects of mathematics is derived from a metaphysical question that arises within the context of Aristotle's theory of substance. "Abstraction", which constitutes Aristotle's view of mathematical objects, does not simply answer the question of the existence and formation of these objects but refers to the specific question of their substantiality. "Abstraction" is Aristotle's answer to the question of whether mathematical objects are substances or not.¹⁷

¹⁶ Bonitz, *Index Aristotelicus* (1960), p. 126b10-25.

¹⁷ The question of the substantiality of mathematical objects is often conflated with the question of their existence. This approach dominates Annas's interpretation of the objective of *Metaphysics* XIII and XIV, according to which these books deal with the

After rejecting the view that mathematical objects are embodied in particular things, Aristotle considers whether mathematical objects are substances. His answer is summarized in the following passage:

They are prior in formula, but not all things that are prior in formula are prior in substance. For those things are prior in substance which when separated from other things continue to exist, but those are prior in formula out of whose formula the formulae of other things is compounded; and these two properties are not coextensive. For if attributes, such as moving or white, do not exist apart from their substances, the white is prior to white man in formula, but not in substance, for it cannot exist separately ... (1077a36-b8)

Considering the question of the substantiality of mathematical objects, Aristotle appeals here to separability as the criterion for substantiality. Employing this criterion, he argues that since mathematical objects are attributes belonging to a primary substance such as “man” they cannot exist separately. The view that Aristotle adopts here was already put forward in the *Categories*; mathematics, as the science of quantities, deals with one of the non-substantial categories, i.e., the category of quantity. Quantity, like the other nine categories, does not mark a self-subsistent being; it is a certain predicate and therefore it depends on objects belonging to the category of substance. So as regards ontology, the question of the substantiality of mathematical objects can be answered definitively: the objects of mathematics are attributes and are therefore not substances. However, Aristotle ends the discussion in *Metaphysics* XIII.2 with a less conclusive remark: “the objects of mathematics are not more substances than the perceptible substances” (1077b12-13). Although mathematical objects are attributes, Aristotle does not utterly dismiss their substantiality.

Aristotle’s ambiguous attitude to the substantiality of mathematical objects is an expression of his ambiguous attitude to attributes in general. In *Metaphysics* VII.4 he claims that the essence is the main mark of a substance since it is the sole entity that can be defined (1030a11-12). Yet immediately after establishing this contention Aristotle modifies it, stating that “definition” has various meanings that correspond to the meanings conveyed by the other categories of being (1030a17-27). Hence attributes, belonging to categories such as quality and quantity, are also definable. Aristotle’s modification of the contention that only substances can be defined deviates from the view presented in the *Categories*. The reasons for this modification

question of whether mathematical objects exist. Annas, *Aristotle’s Metaphysics: Books M & N* (1976) p. 27ff. The impropriety of this view becomes clearer in the sequel.

are discussed in the next section. Here I try to explain Aristotle's notion of abstraction in light of his general attitude to the definability of attributes.

The Greek word *aphairesis* means subtraction or "taking away". Considering mathematical objects as "the results of abstraction", Aristotle suggests that their formation requires a certain act of elimination. Yet Aristotle's examples of this procedure leave open the question of what is eliminated in the procedure of abstraction. For instance, in the *Posterior Analytics* I.5 the act of abstraction involves both eliminating matter, such as bronze, as well as attributes, such as isosceles (74a37-b1). As Mueller points out, eliminating matter and eliminating attributes give rise to different conceptions of mathematical objects. By eliminating matter, mathematical objects are conceived of as universal concepts, such as triangularity, whereas by eliminating attributes, mathematical objects are considered concrete objects, lacking several properties.¹⁸ Aristotle's discussion in *Metaphysics* XIII.2 sheds light on this problem. Immediately after the above-quoted answer to the question of the substantiality of mathematical objects, Aristotle says:

For it cannot exist separately, but always along with the compound thing; and by the compound thing I mean the white man. Therefore it is plain that neither is the result of abstraction prior nor that which is produced by adding posterior; for it is by adding to the white that we speak of the white man. (1077b9-11)

The first thing to be noticed about this passage is that Aristotle considers the basic element from which mathematical objects are derived to be a composite of subject and attribute, such as "white man".¹⁹ The second thing is that abstraction and addition are presented here as correlative methods that refer to a composite of subject and attribute. Having understood the methods of addition and abstraction as referring to such a composite of subject and attribute, the question of what is being added and eliminated in the process of these methods can be answered. "Addition" means adding the subject of

¹⁸ Mueller, "Aristotle on Geometrical Objects" (1970), pp. 161-162.

¹⁹ Some commentators have been puzzled by the use of the dative *tô leukô* – "to the white" – at line 1077b11 [see Annas, *Aristotle's Metaphysics: Books M & N* (1976), p. 147]. Their perplexity stems from the assumption that the method of abstraction is applied to primary substances. Mueller's contention that Aristotle views abstraction as a method by which both matter and attributes are eliminated rests on this assumption. It is endorsed by other commentators as well. See, for instance, Lear, "Aristotle's Philosophy of Mathematics" (1982), p. 163; Halper, "Some Problems in Aristotle's Mathematical Ontology" (1989), pp. 261-268. However, this passage, as well as Aristotle's discussion of "addition" in *Metaphysics* VII.5, indicate that these methods are applied to composites of subject and predicate; there is nothing puzzling in Aristotle's use of the dative case at this line.

predication, for instance adding “man” (i.e., substance) to the attribute “white”. In light of the correlation between addition and abstraction suggested in this passage, it might be concluded that the term “abstraction” means eliminating the subject of predication. Aristotle’s discussion of composites of subject and attribute in *Metaphysics* VII.5, clarifies what adding or eliminating the subject of predication means.

In *Metaphysics* VII.5 the term addition characterizes the method by which composites of subject and attribute are defined. In that chapter Aristotle analyzes the problem of defining such composites, saying that they should be elucidated by addition (1030b16). He introduces two examples: (1) snub-nose and (2) white-Callias. The snub-nose example stands for composites of subject and attribute, which are related to each other inherently. In this case, Aristotle maintains that the attribute “snub” ought to be defined with reference to its subject “nose”. In the latter case, “white-Callias”, the subject and its attribute are not inherently related. Therefore “whiteness” can be defined without reference to the subject “Callias”. In light of this discussion the meaning of the term addition can be refined; the procedure of addition is a method of definition in which the subject of predication is taken into account, while defining its attribute. Likewise, abstraction may be construed as a method of definition in which the subject of predication is *not* taken into account.²⁰

The paradigmatic case “snub-nose” is introduced in *Metaphysics* VI.1 in connection with the procedure of distinguishing the objects of physics from the objects of mathematics:

Of things defined, i.e. of essences, some are like snub, and some are like concave. And these differ because snub is bound up with matter (for what is snub is a concave nose), while concavity is without perceptible matter. (1025b30-34)

The distinction between snubness and concavity seems to correspond to the distinction between “snub-nose” and “white-Callias”.²¹ In both cases the distinction is introduced to characterize the same method of defining an

²⁰ Since late antiquity, Aristotle’s notion of abstraction has been interpreted as an epistemological process by which we grasp universals through sense perception. On this see, Mueller, “Aristotle’s doctrine of Abstraction in the Commentators” (1990). Cleary criticizes this view, arguing that abstraction is a logical method by which the primary subject of predication is isolated [Cleary, “On the Terminology of Abstraction in Aristotle” (1985)]. I agree with Cleary that abstraction is a certain logical method. Yet Cleary’s interpretation fails to account for the parallelism between abstraction and addition, since in both procedures a certain subject of predication is isolated. When both these procedures are understood to be methods of definition, this parallelism is explained.

²¹ Notice that in *Metaphysics* XIII.2 1077b9 Aristotle uses a similar example i.e., “white man”, in discussing mathematical objects.

attribute. In defining snubness, one should take into account its subject, “nose”, whereas “concavity” can be defined without reference to perceptible subjects. “Snubness” and “concavity” exemplify the objects of physics and mathematics respectively; the objects of physics as well as the objects of mathematics are compounds of attributes and perceptible substances. These attributes differ in the relation that subsists between them and their subject; the objects of physics are attributes, related inherently to a perceptible substance, whereas the objects of mathematics are non-inherent attributes of the perceptible substance. Consequently in *Physics* II.2 Aristotle argues that the objects of physics are less separable than the objects of mathematics (193b36-194a1).

Still, the method of abstraction does not involve outright elimination of the subject of predication. In *Metaphysics* VI.1 Aristotle maintains that concavity is without perceptible matter. However, the lack of *perceptible* matter does not imply that mathematical objects have no reference whatsoever to the primary substance. This view is stated explicitly in *Metaphysics* XI.3:

...the mathematician investigates abstractions (for in his investigation he eliminates all the perceptible qualities, e.g. weight and lightness, hardness and its contrary ... and leaves only the quantitative and the continuous... (1061a28-33)

Abstraction involves elimination of the perceptible aspects belonging to the subject of predication, while it maintains the non-perceptible aspects of the subject, such as quantity. Accordingly, the results of abstraction are not universal properties detached from their subject of predication but they remain, after abstraction, attributes of a subject. Aristotle’s discussion in *Metaphysics* XIII.3 makes it clear that mathematical objects are related to a certain aspect of the primary substance:

...there should also be both formulae and demonstrations about sensible magnitudes, not however *qua* sensible but *qua* possessed of certain definite qualities. (1077b20-22)

Mathematical objects are defined with reference to the perceptible substance yet they are not defined with reference to the perceptible *aspect* of the substance but with reference to a certain non-perceptible aspect of it.

This interpretation of Aristotle’s notions of abstraction and addition results in the conclusion that abstraction and addition are methods of defining composites of subject and attribute. When abstraction is viewed as a method of definition, mathematical objects, which are “the results of abstraction”, are in fact definitions, i.e., universal concepts. The results of abstraction, then, are not particular mathematical objects, such as a triangle,

but mathematical concepts such as “triangularity”. To corroborate this conclusion further, I discuss Aristotle’s view of the matter of mathematical objects.

In *Metaphysics* VIII.6 Aristotle employs the distinction between form and matter in accounting for the unity of a definition: “But as we say, one element is matter and another is form, and one is potentiality and the other actuality” (1045a23-24). Definitions have a material aspect, yet this material aspect, I argue, is not identical with the matter of the concrete object but is analogous to the concrete matter. In *Metaphysics* VII.12 Aristotle maintains that the genus exists like matter, since it cannot exist apart from the species subsumed under it (1038a5-8). The material aspect of a definition, then, is not the matter that constitutes the particular object. The matter of definition is a universal concept, a genus. The analogy between genus and matter holds for their mode of existence; just as unformed matter is merely a potential being, genera have a potential being as long as they are not specified by their *differentiae*.²² In *Metaphysics* V.28 matter is said to be one of the senses of the term “genus”; clarifying this sense, Aristotle appeals to mathematical examples:

There are kinds in the sense in which plane is the kind of plane figures and solid of solids ... and this is what underlies the *differentiae*. (1024a36-b4)

The matter of the defined mathematical objects is a genus, e.g., the universal concept of plane figures or solid figures. A specific geometrical object, such as a triangle, results from a combination of the genus and the *differentiae*. For instance, a triangle is a certain plane figure.

The material aspect of the definition should not be confused with the intelligible matter that Aristotle introduces in connection with mathematical objects (1036a9-12 and 1037a2-5). In *Metaphysics* VII.10 he poses the question of whether the material parts of an entity correspond to the parts of its definition. Aristotle asserts that the material parts of the concrete entity are not part of its definition.²³

²² In his “Genus as Matter” (1973), Rorty claims that Aristotle identifies the genus with matter. In my opinion the textual evidence supports only an analogical relation between these notions. Further, understanding matter as identical with the genus does not accord with Aristotle’s notion of definition. Species are defined by the genus and the *differentiae*; the particular object, by contrast, is defined with reference to its species, whereas its material elements are not part of its definition (1035a20). Rorty’s view cannot explain how the genus, which is considered identical with matter, is a part of a definition.

²³ Mueller, “Aristotle on Geometrical Objects” (1970), argues that Aristotle conceives of mathematical objects as particular objects since they have an intelligible matter. Mueller identifies the intelligible matter of mathematical objects with the genus, which is part of the definition of these objects (*ibid.*, p. 167). However, intelligible matter is part of the

Therefore the clay statue is resolved into clay ... and again the circle into its segments; for there is a sense of 'circle' in which it involves matter. For 'circle' is used homonymously, meaning both the circle in general and the individual circle, because there is no name proper to the individuals. (1035a31-b3)

This passage suggests that the matter of the geometrical figure is the matter of the particular object and not the genus, which is the matter of the defining formula. In the sequel, Aristotle identifies the matter of the particular geometrical figure with intelligible matter:

I mean by intelligible circles the mathematical, and by sensible circles those of bronze and of wood, of these there is no definition ... When they go out of our actual consciousness it is not clear whether they exist or not And some matter is sensible and some intelligible, sensible matter being for instance bronze and wood ... and intelligible matter being that which is present in the sensible but not *qua* sensible, i.e., in the objects of mathematics. (1036a3-12)

Here Aristotle draws an analogy between intelligible matter and sensible matter. According to this analogy, intelligible matter, like sensible matter, belongs to *particular* mathematical objects. Aristotle claims, further, that a particular mathematical object, like any other particular object cannot be defined or known, although it is endowed with intelligible matter. Consequently, Aristotle's notion of intelligible matter has no bearing on the objects of mathematics as "the results of abstraction". Abstraction is a method of definition; it does not refer to intelligible matter as the material ingredient of the definition; rather the genus, which is a universal concept, serves as this material ingredient. In construing the objects of mathematics as results of abstraction, Aristotle conceives of mathematical objects as universal concepts. Having discussed the status of mathematical objects as the results of abstraction, I turn now to consider the relationship between these objects and syllogistic reasoning.

Aristotle's attitude to the objects of mathematics in particular, and to composites of subject and attribute in general, involves expansion of the range of applicability of definitions to non-substantial categories. In *Metaphysics* VII.4, Aristotle poses the following question:

But since there are compounds of substance with the other categories ... we must inquire whether there is a formula of the

particular object, while the genus is part of the definition. In the above quoted passage Aristotle explicitly denies any correspondence between these two aspects of substances.

essence of each of them, i.e. whether to these compounds also there belongs an essence, e.g. to “white man”. (1029b22-27)

At the beginning of the discussion of composites of subject and attributes, this question of whether such composites can be defined is answered negatively; Aristotle concludes that only species can be defined (1030a11-12). However, as I said, shortly afterwards Aristotle modifies this contention, claiming that “definition” has several meanings that correspond to the ten categories of being (1030a17-27). Although definition and essence are, according to *Metaphysics* VII, the main indicators of a substance, Aristotle assigns these characteristics to attributes as well. Since mathematical objects are a special case of subject-attribute composites, their characterization as results of abstraction implies that they are defined composites of subject and attribute (1077b20-22). Abstraction and addition, then, involve treating such composites as quasi-substances. By means of these procedures a composite of subject and attribute is regarded as having two of the characteristics of a substance: separability and essence. The formation of mathematical objects by means of the method of abstraction requires detaching the attribute from its subject and defining it according to its genus and *differentiae*. Endorsement of the procedures of abstraction and addition renders vague the sharp distinction between substance and attribute; the priority of primary substances cannot be explained solely in terms of definition or essence for these characteristic indicators of the primary substance are held to belong to non-substantial entities (e.g., quantities) also, despite their secondary ontological status.²⁴

Aristotle’s attitude to the objects of mathematics is an expression of his attitude to the objects of knowledge in general. In *Metaphysics* VI.1 Aristotle distinguishes between first philosophy, i.e., metaphysics, and demonstrative knowledge. The former, according to Aristotle, deals with being *qua* being, whereas the latter deals with some particular genus of being. In this discussion the genera of being, which are dealt with by *demonstrative knowledge*, are exemplified by the paradigmatic expressions “snubness” and “concavity”. Hence the objects of demonstrative knowledge, like the objects of mathematics, are composites of subject and attribute.²⁵ Retaining the

²⁴ This claim is confined to Aristotle’s discussion of substance in *Metaphysics* VII.4, in which he attempts to distinguish between substances from attributes in terms definitions and essence. The endorsement of abstraction and addition as methods of definition does not affect the other characteristics of a substance, such as being a “this” (*tode ti*), being self-subsistent, or being a primary subject of predication.

²⁵ In *Metaphysics* VI.1 (1025b34) Aristotle says that the subject matter of all natural science is analogous to “snubness” (*panta ta physica homoiôs tō simō legontai*). Since Aristotle exemplifies this contention by terms such as nose, eye, animal, and plant, it seems that all natural sciences, including biology, treat their subject matter as quasi-substances.

sharp distinction between substances and attributes, according to which essence and definition belong to substances alone, would exclude these composites of subject and attribute from being objects of knowledge. Indeed, in the *Posterior Analytics* I.13, Aristotle conceives of the substantialization of attributes as indispensable for demonstrative knowledge. In that chapter he asserts that although geometrical objects may be predicated of an underlying subject, they are studied as if they are not predicated of an underlying subject (79a8-10).

Consequently, by providing an account of the definability of composites of subject and attribute, the method of abstraction secures the status of mathematical objects as objects of *demonstrative* knowledge. The use of this method enables these objects to serve as terms in syllogistic inferences. In my analysis of syllogistic inferences I argued that syllogistic reasoning depends upon conceptual relations between the syllogistic terms. Once the substantialization of the object of knowledge is disapproved, objects of knowledge cannot be considered to have the essential attributes required for carrying out syllogistic inferences. Lacking an essence, objects are nothing but a collection of material elements. The relations between these elements are external since there is no intrinsic nature that determines the relations between them. To accommodate the objects of mathematics to his theory of knowledge and to syllogistic reasoning, Aristotle treats these objects as quasi-substances.

This analysis of the relationship between the theory of demonstration and Aristotle's view of the objects of knowledge traces the reasons for the discrepancy between Aristotle's theory of knowledge and scientific practice to his metaphysics. Indeed, Aristotle's treatment of mathematical examples throughout the *Posterior Analytics* manifests an inherent discrepancy between his theory of knowledge and scientific practice which renders Aristotle's theory of knowledge deficient only on the assumption that it is aimed at providing an adequate analysis of scientific practice. But on the assumption established in the foregoing discussion, that Aristotle's theory of knowledge is part of his general metaphysical views, the discrepancy between the theory of demonstration and scientific practice is no longer a major shortcoming of Aristotle's theory. His attitude to scientific practice is normative and so the scientific examples scattered throughout the *Posterior Analytics* pose a problem that cannot be settled. Nevertheless, regardless of the question of the adequacy of these examples, Aristotle's theory of knowledge is an integral part of his philosophical system. It coheres with his metaphysics, more specifically with his notion of substance. Aristotle's attempt to accommodate the objects of knowledge to the notion of substance explains the prominence of syllogistic reasoning in his theory of

demonstration. An analysis of the scientific practice of Aristotle's contemporaries adds support to the contention that it is the notion of substance and not scientific practice that serves as the theoretical foundation of the theory of demonstration.

4.5 GREEK MATHEMATICAL REASONING

According to Proclus, a Euclidean proof has six parts: (1) *protasis*: enunciation, i.e., a general statement of the theorem to be proved, introducing in the perfect cases both the given and the sought; (2) *ekthesis*: setting out the given by referring to a particular instance, which is described in the diagram; (3) *diorismos*: a restatement of that which is sought in terms of the particular instance; (4) *kataskeuê*: construction, i.e., the addition of something wanting in the given, in order to find that which is sought; (5) *apodeixis*: proof; (6) *sumperasma*: the statement of the conclusion in terms of the particular instance.²⁶ In a Euclidean proof, then, that which is sought is stated three times: first it is formulated generally in the *protasis*, then it is stated in particular terms in the *diorismos*, and finally it is stated again in particular terms in the *sumperasma*. Euclid's proof is carried out on the basis of the *ekthesis-diorismos* stages, i.e., the statement of the theorem in particular terms, and yet it proves the general proposition, introduced at the *protasis* stage. Although the proof itself concerns a particular instance, it nevertheless establishes a general conclusion.

From a strictly logical point of view such a proof cannot be regarded either as a deduction or as an induction; a general conclusion does not follow deductively from particular premisses, while induction cannot be based on one instance. Nevertheless, Euclid's proof is not deficient since it yields the conviction that a proof concerning a particular instance actually establishes the general conclusion. Commenting on geometrical conclusions, Proclus contends that mathematicians draw a double conclusion; on the one hand, the conclusion is shown to be true of a particular instance, and on the other, it is true in general. Proclus justifies this procedure, claiming that the particular figures resemble other figures of the same sort.²⁷ Similarly, Kant points to the special character of mathematical objects, noting that "mathematical knowledge considers the universal in the particular".²⁸ The distinction between the universal and the particular, which is fundamental to Aristotle's philosophy, appears inapplicable to Greek mathematics. The

²⁶ Proclus, *In Primum Euclidis Elementorum Librum Commentarii*, pp. 208-210.

²⁷ *Ibid.*, 207.

²⁸ Kant, *Kritik der reinen Vernunft* (1913), p. 600 [742.20-21].

unique character of mathematical objects, which are neither universals nor particulars, is explained through a comparison between mathematical concepts and generic concepts.

Generic concepts are formed from the sensible multitude, by selection from the wealth of particular objects the features that are common to several of them. The formation of generic concepts consists in selecting from the plurality of objects only the similar properties, while neglecting the rest. In this procedure what is merely a part of the sensuous whole is regarded as characterizing and explaining the whole. From the outset the generic concept neglects the unique characteristics of particular instances. The features that distinguish one particular instance from another are not included in the generic concept. Therefore, a general conclusion concerning all the instances cannot be drawn from a particular instance, considered in its particularity; for consideration of a particular instance involves an examination of the very features that the generic concept ignores.

In contrast to generic concepts, individuation appears irrelevant to mathematical concepts. From a mathematical point of view one circle does not differ from another. Concerning its mathematical properties, a particular mathematical object is identical with another one. A particular mathematical object is viewed as a paradigmatic-particular; proving that a particular triangle has a certain property indicates that all the other triangles have the same property. However, the formation of mathematical objects does ignore certain aspects of the particular object. In a geometrical proof a mathematical object such as a triangle is not considered in its particularity. For instance, geometrical investigation ignores the triangle's magnitude or the angles' magnitude. Although mathematical concepts, like generic concepts, ignore certain aspects of the particular instances, a proof concerning a particular triangle counts as a valid proof concerning any triangle whatsoever. Since both mathematical concepts and generic concepts are formed by ignoring certain features of particular instances, it might be asked why in the former case a particular instance serves as a paradigm of all other instances, whereas in the latter case the particular instance retains its particularity. An analysis of Euclid's first three postulates reveals the difference between mathematical and generic concepts; mathematical concepts are abstracted from operations, while generic concepts are abstracted from objects.

Laying down the foundation of the *Elements*, Euclid enunciates the following postulates: (1) to draw a straight line from any point to any point; (2) to produce a finite straight line continuously in a straight line; (3) to describe a circle with any center and distance. These construction postulates are commonly interpreted as existence assertions: they imply the existence

of a straight line, an infinite straight line, and a circle. Yet constructibility and existence give rise to different conceptions of geometrical objects. Postulating construction means that the existence of the constructed object is not assumed in advance. As Mueller points out, Euclid's geometry does not assume an underlying system of objects but basic operations that yield the objects of mathematics.²⁹

The priority of construction over existence indicates that mathematical concepts are not formed in the same way as generic concepts. The objects of geometry are not obtained through an idealization that eliminates certain properties from a given sensible object. They are results of construction. The properties of geometrical objects are determined by the constructions that generate them. Properties that do not require a special act of construction are not regarded as relevant to geometrical inquiry. For instance, the magnitude of a line is irrelevant from a geometrical point of view since the same construction is required regardless of the size of the drawn line. Similarly, the magnitude of an angle is irrelevant to geometrical analysis, since constructing angles of different sizes does not require different constructing means. So although the particular geometrical figures are different, a particular instance can serve as a paradigm since geometry investigates the features of the object insofar as it is constructed. Individual differences between specific geometrical figures, which do not find expression in different constructions, do not affect the universality of geometrical proofs.

The status of geometrical objects having been determined as results of construction, the theoretical presuppositions of Euclid's practice can be exposed. Euclid's construction postulates are formulated in Hilbert's *Grundlagen* as existential propositions. For instance, Euclid's first postulate is formulated as follows: "For every two points A, B there exists no more than one line that contains each of the points A, B".³⁰ Similarly, geometrical problems, such as the problem of constructing an equilateral triangle (*Elements*, I.1), might be interpreted as existence proofs, for instance, of an equilateral triangle.³¹ Yet a comparison between Euclid's first postulate and

²⁹ Mueller, *Philosophy of Mathematics and Deductive Structure in Euclid's Elements* (1981), p. 14.

³⁰ Hilbert, *Foundation of Geometry* (1971), p. 3.

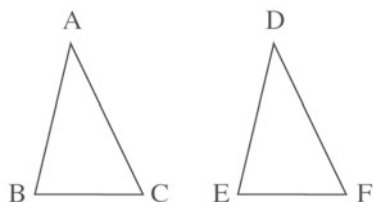
³¹ The interpretation of Euclid's constructions as existence claims was first introduced in Zeuthen, "Geometrische Konstruktion als Existenzbeweis in der antiken Geometrie" (1896), pp. 272-278. Since then this interpretation has been elaborated by other historians of Greek mathematics, and has been adopted by those of Aristotle's commentators who endorse a parallelism between Euclid's axiomatics and Aristotle's theory of demonstration. For a criticism of the existential interpretation of Greek geometrical constructions see Knorr, "Constructions as Existence Proofs in Ancient Geometry" (1983), and Harari, "The Concept of Existence and the Role of Constructions in Euclid's *Elements*" (2003).

Hilbert's first axiom indicates that the logical role of construction is equivalent not to the assumptions of existence but to an explicit statement of spatial relations.

Hilbert's *Grundlagen* opens with a specification of three sets of objects: points, lines, and planes. These objects are not explicitly defined and their meaning is determined by spatial relations, such as "lie", "between", and "congruent", which subsist between them. In Hilbert's geometry, as in Euclid's *Elements*, geometrical objects are not laid down at the outset but are derived from certain relations that subsist between them. A point, for instance, is nothing but the object that satisfies the axioms of incidence. In Hilbert's *Grundlagen*, as well as in Euclid's *Elements*, geometrical objects are considered results of antecedent assumptions; in the former, geometrical objects are derived from certain relations, whereas in the latter they are results of construction. Accordingly, Euclid's construction postulates are equivalent to Hilbert's axioms of relations, while the existence of the geometrical figures is in both cases a derived conclusion.

Despite the equivalence between Euclid's *Elements* and Hilbert's *Grundlagen*, these two geometrical theories are based on different presuppositions. Taking the relations of incidence, order, congruence, and continuity as basic concepts, Hilbert forms a system of spatial relations and applies them to a given set of geometrical objects. The existence of geometrical objects is asserted explicitly as part of the characterization of the spatial system. By contrast, Euclid does not appeal to general terms that denote relations; he characterizes the objects by specific constructions that logically correspond to the general relation. For instance, the relation of incidence is not expressed explicitly but is exhibited, for instance, through the intersection of two circles. Generally, Euclid does not conceptualize spatial relations but makes them plain through actual constructions. Since Euclid's geometry is based on operations that exemplify spatial relations, geometrical objects are not viewed as preexisting entities but are objects that need to be brought into existence.

Euclid's handling of congruence is the most conspicuous expression of his non-conceptual attitude to spatial relations. Unlike Hilbert, Euclid does not lay down the first congruence proposition as an axiom. In *Elements* I.4, he proves: if two triangles have the two sides equal to two sides respectively, while the angles contained by the equal straight lines are equal as well, the two triangles are congruent.



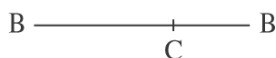
In proving the first theorem of congruence, Euclid conceives of one triangle (DEF) as being laid over another triangle (ABC), so that point D is placed upon point A and the side DE is placed upon the side AB. Laying one triangle over another triangle in this manner leads to the conclusion that the base of triangle DEF falls on the base of triangle ABC. In the assumption that the angles contained between the two pairs of equal sides AB, AC and DE, DF are equal, it follows that the other angles of the two triangles fall upon each other. Euclid's proof rests on the act of placing one figure upon another figure, which implies that congruence is conceived of as equality. In this proof the relation of congruence is not conceptualized but is exhibited through construction, i.e., by the act of placing one figure upon another.³² Consequently, Euclid's geometry does not abstract the spatial relations out of the act of construction; he employs constructions for the sake of exhibiting these spatial relations.

Such a non-conceptual attitude to mathematical relations characterizes Euclid's arithmetic as well. Book VII of the *Elements* opens by defining a unit: "A unit is that in virtue of which each of the things that exist are called one". Taking the unit as the fundamental concept of his arithmetic, Euclid defines a number as "a multitude composed of units". This definition has two consequences that reflect the operative character of Euclid's arithmetic. The first obvious consequence of this definition is that a unit is not a number. The exclusion of the integer "one" from Euclid's arithmetic indicates that the notion of a series of natural numbers is absent from his arithmetic. In modern arithmetic, the positive integers are obtained by successive addition of the integer "one". Accordingly, numbers can be defined as having a relation of succession to each other; 2 is the successor of 1, 3 is the successor of 2, and so forth. As a result, numbers are conceived of as having mutual relations of succession, i.e., relations of before and after. By contrast, Euclid's definition of a number does not provide a general

³² Von Fritz points out that in the *Elements*, the Greek verb *epharmozein* (to coincide) never appears in the general theorems, but only in the proofs themselves. However, according to von Fritz, even in the proofs the verb *epharmozein* is not used as a technical term: von Fritz, "Gleichheit, Kongruenz und Ähnlichkeit" (1971), p. 431. The non-technical use of the term "congruence" indicates that Euclid does not conceptualize this relation.

account of the relations of succession that subsist between numbers. In Euclid's definition, each number is a certain collection of units having no explicit relation to other collections of units. The second consequence of Euclid's definition of number is that this definition tacitly implies the relations of succession. Taken by itself, the definition "a number is a multitude composed of units" does not yield the concept of number unless this multitude of units is counted. Without counting, this multitude of units is not in itself a number but a collection of objects. It becomes a number as a result of counting these units. Euclid's concept of number does not entirely lack the relation of succession. This relation is assumed implicitly, since Euclid's definition of a number presupposes the act of counting.³³

Euclid's arithmetic, like his geometry, does not conceptualize arithmetical relations. Rather, it is the implicit act of counting that may play the logical role of an explicit definition of arithmetical relations. Euclid's arithmetical books offer scarcely any articulated definition of mathematical operations such as addition and subtraction.³⁴ Yet implicit references to such operations are to be found in the arithmetical proofs themselves. In these cases, the arithmetical operations are not done in the abstract but with reference to diagrammatic representations of numbers. For instance, in *Elements* IX.24 Euclid proves that if an even number is subtracted from an even number, the remainder will be even.



Referring to this diagram, Euclid says: "For from the even number AB let the even number BC be subtracted: I say that the remainder CA is even". In the absence of a general definition of subtraction, Euclid's proof requires the reader to subtract a specific number, represented by line BC, from another specific number, represented by line AB. This proof requires the imaginary act of taking line CB from line AB. Generally, Euclid does not introduce operation symbols, such as +, nor does he employ variables for designating numbers. Operation symbols and variables presuppose arithmetical relations. Operation symbols symbolize defined arithmetical operations, which characterize a certain relation between numbers. Similarly, the employment of variables assumes the possibility of ignoring the specific values of numbers. This assumption can be justified if specific numbers are conceived

³³ Klein shows that the Greek concept of number presupposes counting. According to Klein's analysis, in Greek mathematics the concept of number is conceived of as always related to numbered objects: Klein, *Greek Mathematical Thought and the Origin of Algebra* (1968), pp. 46-60.

³⁴ The only operation involving numbers that Euclid explicitly defines is multiplication.

of as a special case of a general principle that specifies the relation of succession. Consequently, Euclid's mathematics is based on an intuitive acquaintance with mathematical relations. Since these relations have not been conceptualized, concrete operations of construction are required for formulating Euclidean proofs.³⁵

In light of this account the discrepancy between mathematical practice and Aristotle's logic can be explained as rooted in the basic presuppositions of Aristotle's metaphysics. The notions of substance and essence are introduced in accounting for the unity of objects, as well as for the causal relations in nature. Aristotle's physics appeal to matter as a constituent of nature, yet the material ingredients are not conceived of as mere heaps (*soroi*) since they are regarded as a means of attaining the end, which is the essence. Hence, the essence determines the unity of the material elements by providing a teleological explanation for their being. For instance, bricks and stones form a house, since they are assembled with the aim of providing shelter. Aristotle adopts a similar attitude to the objects of mathematics. In *Metaphysics* VIII.3 he says:

And the number must have something in virtue of which it is one thing, while our opponents cannot say if indeed it is one (for either it is not one but a sort of heap, or if it is, we ought to say what it is that makes one out of the many). (1044a2-5)

This passage expresses Aristotle's criticisms of the Greek conception of number. From Aristotle's point of view, the definition of number as a collection of units is deficient, since it cannot account for the unity of number. In the previous section I showed that Aristotle applies his theory of substance to mathematical objects. Thus, he introduces his notions of essence and substance when addressing the question of the unity of mathematical objects. This attitude finds expression in Aristotle's attempt to classify these objects under genera and species:

It is said rightly, too, that the number of the sheep and of the dogs is the same number if the two numbers are equal, but not the same decade or the same ten; just as the equilateral and the scalene are not the same triangle, yet they are the same figure, because they are both triangles. For things are called the same so-and-so if they do not differ by the differentia of that thing, but not if they do; e.g. triangle differs from triangle by a

³⁵ Unlike the Pythagoreans, Euclid uses lines and not dots in representing numbers. Euclid's representation, then, does not presuppose acts of counting dots but geometrical constructions by which magnitudes are added or subtracted. This mode of representation reflects the priority of geometry over arithmetic in Euclid's *Elements*.

differentia of triangle, therefore they are different triangles; but they do not differ by a differentia of figure, but are in one and the same division of it. (224a2-10)

Aristotle's approach has advantage for understanding physical and biological phenomena, but his essentialism is inadequate for mathematics. As I have shown, in Euclid's *Elements* mathematical objects are results of construction or counting operations, while Aristotle conceives of them as results of abstraction. The generic conceptualization of mathematical objects regards the results of construction, i.e., lines, planes figures, and solids, as belonging to the matter and not to the form. The objects of mathematical knowledge are essences or universals, in which the process of the formation of the particular object does not find expression. Hence, by reducing the process to its essence, Aristotle's essentialistic attitude results in disposing of those operations, which are manifestations of the very relations that mathematics investigates.

Greek mathematical practice shares with Aristotle the non-conceptual attitude to spatial relations. Neither Aristotle nor Euclid provides a theoretical account of such relations. Yet Euclid, unlike Aristotle, does not dispose of mathematical relations but appeals to the intuitive understanding of them. By contrast, Aristotle's attitude to mathematics attempts to detach mathematical reasoning from intuitive understanding, while offering no conceptual account of relations. Accordingly, Aristotle's theory of demonstration, which takes substance and essence as its first principle, does not accommodate Greek mathematical practice. In the next chapter I show that Aristotle's concept of knowledge is formed in opposition to the perceptual-intuitive view that underlies Greek mathematical practice.

Chapter Five

Knowledge and Demonstration

5.1 KNOWLEDGE AND UNDERSTANDING

In the *Posterior Analytics* I.2 *epistêmê* – “knowledge” – is defined as follows:

We think we know (*epistasthai*) a thing *simpliciter* ... whenever we think that we know (*ginôskein*) that the cause, due to which the thing is, is its cause, and that it is not possible for this to be otherwise. (71b9-2)

This definition intertwines two notions that are commonly considered separate in modern thought. In the first part of the definition *epistêmê* is defined as an acquaintance with the cause, i.e., with the explanation, while in the second part of the definition *episteme* is defined as an acquaintance with the modal status of the known content. According to this definition, having *epistêmê* of X means that one knows that X’s explanation is indeed its explanation and that one knows that X is necessary. From a modern point of view, each of these parts of Aristotle’s definition gives rise to two different interpretations of the term *epistêmê*; by referring to causes, the first part characterizes *epistêmê* as explanation and understanding, while the second alludes to justification and knowledge, by stressing the modal status of the known content.

However, understanding or explanation and justification or knowledge cannot be included in one and the same concept, for understanding and explanation presuppose knowledge and justification. Explanation yields understanding of certain facts; hence, it proceeds from the assumption that the explained facts are already known. Justification, by contrast, yields knowledge by establishing certain facts; hence, prior to justification, the facts are considered to be unknown. So if Aristotle’s notion of *epistêmê* is understood to comprise both knowledge and explanation, the demonstrative syllogism would be rendered circular. On the assumption that demonstration yields both knowledge and explanation, the demonstrative premisses are related to the conclusion in two incompatible ways; on the one hand, if

demonstration is regarded as a means of justification, the demonstrative premisses establish the conclusion. In this case the premisses refer to the conclusion as yet unknown. On the other hand, if demonstration is regarded as a means of explanation, the demonstrative premisses explain the conclusion. However, in this case the premisses refer to the conclusion as already known. Hence, interpreting Aristotle's notion of demonstrative knowledge as a theory of knowledge *and* explanation leads to a sheer contradiction; for in this interpretation, the conclusion would be both unknown and already known, at one and the same time.

To avoid this contradiction Aristotle's modern commentators stress one aspect of Aristotle's definition of *epistêmê* while neglecting the other. Focusing on the second part of Aristotle's definition, the traditional interpretation construes the Greek term *epistêmê* as "scientific knowledge" and regards the demonstrative syllogism as a means of justification.¹ The first elaborate exposition of this interpretation was provided by Scholz, who views Aristotle's theory of demonstration as dealing with the axiomatization of science. According to Scholz's interpretation, Aristotle conceives of *epistêmê* as having the following characteristics: (1) it is a sequence of sentences; (2) these sentences are divided into primitive sentences, i.e., axioms, and derived sentences, i.e., theorems; (3) the terms appearing in these sentences are divided into primitive terms and defined terms; (4) the primitive sentences are immediately evident and indemonstrable; (5) apart from these and from the rules of logic, nothing else is required to prove a theorem; (6) the primitive terms are indefinable; (7) apart from these and certain conjunctive operations, nothing else is required for the construction of the derived terms.² Scholz, then, views *epistêmê* as knowledge, which is attained through a deductive procedure that yields justification.

However, as the above quoted definition of *epistêmê* indicates, truth and certainty are not the sole requirements that Aristotle imposes on demonstrative knowledge. Consequently, the axiomatic interpretation of the *Posterior Analytics* has been criticized for ignoring the explanatory characteristics of *epistêmê*. Opponents of the axiomatic interpretation, such as Kosman and Burnyeat, claim that *epistêmê* has to be rendered as "understanding", since according to Aristotle demonstration should proceed from principles that are more familiar than the conclusion, are prior to the conclusion, and are the causes of the conclusion (71b29-30). Accordingly, it

¹ This interpretation is traditional and can be found in many discussions of the history of philosophy. I mention just a few of its proponents here: Ross, *Aristotle's Prior and Posterior Analytics* (1949), pp. 55ff.; Guthrie, *History of Greek Philosophy*, vol. VI (1981), p. 184; Berka, "Aristoteles und die axiomatische Methode" (1961), pp. 200-205; Matten, "The Structure of an Aristotelian Science" (1987), p. 2.

² Scholz, "The Ancient Axiomatic Theory" (1975), pp. 52-53.

has been argued that Aristotle's main concern in the *Posterior Analytics* is to provide a theory of explanation rather than a theory of knowledge and justification.³ Yet in construing *epistêmê* as understanding, the proponents of the explanatory interpretation deviate from the original meaning of the Greek word *epistêmê*, which means skill, acquaintance, and knowledge. This shortcoming extends beyond terminology, for the explanatory view does not take into account Aristotle's intention to provide a theory of knowledge. The explanatory interpretation indicates that Aristotle's notion of *epistêmê* is in fact close to the modern notion of understanding, and distinct from the modern notion of knowledge. However, this interpretation does not simply point to the difference between Aristotle's and the modern notion of knowledge. Instead, this interpretation equates Aristotle's notion of *epistêmê* with "understanding". In so doing, the explanatory interpretation evaluates Aristotle's theory of demonstration in light of modern thought rather than disclosing Aristotle's own concept of knowledge.

The notion of knowledge, applied by Aristotle's modern commentators presupposes the distinction between judgment and understanding. In this view, understanding is the grasp of concepts, meanings, or any other cognitive content. It is indifferent to the truth-value, ontological status, or modal status of the understood content. Judgment, on the other hand, is a decision about the truth-value, ontological status, or modal status of a given content. Since understanding is a grasp of cognitive content, it is common to various cognitive states, such as knowledge, belief, and perception. The difference between these cognitive states lies in the judgment that the subject forms about the understood content. A grasp of a given content would be regarded as knowledge if the subject judged it as necessary, while it would be regarded as a belief if the subject judged it as true. Accordingly, knowledge differs from other cognitive states in the truth-value, modal status, or ontological status that are assigned to a given content. This view excludes understanding, which does not carry with it conviction and certainty from being knowledge.

In their preoccupation with the distinction between judgment and understanding Aristotle's modern commentators attempt to classify his theory of knowledge under one of these notions. By stressing notions such as truth, evidence, certainty, and justification the proponents of the axiomatic interpretation construe *epistêmê* as knowledge. On the other hand, the proponents of the explanatory interpretation view *epistêmê* as understanding, since in their opinion Aristotle's *Posterior Analytics* lacks a judgmental

³ Kosman, "Understanding, Explanation and Insight in the *Posterior Analytics*" (1973); Burnyeat, "Aristotle on Understanding Knowledge" (1981), Barnes, *Aristotle's Posterior Analytics* (1993), p. 82.

aspect. Both interpretations read the modern notion of knowledge into Aristotle's theory of demonstration. The proponents of the axiomatic interpretation, who stress the judgmental aspect of knowledge, claim that Aristotle's notion of knowledge accords with the modern notion of knowledge. By contrast, the proponents of the explanatory interpretation, who stress the differences between Aristotle's notion of knowledge and the modern notion of knowledge, do not regard *epistêmê* as knowledge at all.

Having interpreted Aristotle's logic as a logic of understanding, I agree with the proponents of the explanatory view regarding the absence of a judgmental aspect from Aristotle's theory of knowledge. But I disagree with this interpretation insofar as it characterizes his theory of demonstration as a theory of explanation. Following the common connotation of the Greek word *epistêmê*, I attempt to illumine the presuppositions that led Aristotle to view demonstrative knowledge as understanding. The following discussion of the role of demonstrative syllogism in attaining knowledge indicates that the modern distinction between judgment and understanding is inapplicable to Aristotle's notion of knowledge. I show that Aristotle forms his notion of knowledge in light of a distinction between two types of understanding: perceptual and conceptual. I start with an analysis of the cognitive state prior to demonstration.

5.2 DEFINITION AND BEING

The second book of the *Posterior Analytics* opens with a fourfold classification of the questions asked in demonstrative knowledge. Knowledge, according to Aristotle, involves seeking: (1) the fact (*to hoti*); (2) the reason why (*to dioti*); (3) whether it is (*ei esti*); and (4) what it is (*ti esti*) (89b23-25). These four questions are classified under two groups: (I) concerning attributes, which refer to (1) the fact and (2) the reason why; and (II) concerning substances or grammatical subjects, which refer to (3) "whether it is" and (4) "what it is" (89b26, 32). Further, these four questions belong to different stages in the demonstrative inquiry; the fact (1) and the question "whether it is" (3) are preliminary questions required for answering the other two; namely the reason why (2) and "what it is" (4) (89b29, 34). A discussion of the preliminary stage of demonstration, which concerns the questions of being, elucidates Aristotle's view of the mutual relations between being and definition.

In the *Posterior Analytics* II.8 Aristotle describes how the question of being, i.e., "whether it is", is to be answered:

But as to whether it is, sometimes we grasp it accidentally, and sometimes when grasping something of the object itself – e.g. of thunder, that it is a sort of noise in the clouds; and of eclipse, that it is a sort of privation of light ... (93a21-24)

Setting aside for now Aristotle's distinction between accidental and non-accidental grasp of being, this passage indicates that Aristotle views the question of being as answered by a definition, such as "thunder is a sort of noise in the clouds". This contention gives rise to two queries: first, given that the question of "whether it is" is answered by a definition, how can we maintain Aristotle's distinction between this and the question of "what it is"? Second, how can a verbal account or definition determine being, since non-existent entities can also be defined? Aristotle's modern commentators appeal to his distinction between signification (*ti sêmainei*) and definition (*ti esti*) in accounting for the first query.⁴ According to this view, the definition that answers the question of "whether it is" is not a real definition that specifies the essence of a thing (*ti esti*) but it is a nominal definition; it merely clarifies the meaning of a word by answering the question *ti sêmainei to onoma*. The existential import of the nominal definition is interpreted as an implicit reference to empirical experience of the defined objects. Bolton, for instance, holds that the verbal account that answers the question of "whether it is" gives a loose specification of a certain phenomenon by referring to actual instances that are familiar to us through perception.⁵ Another related view has been suggested by Demoss and Devereux, who state that a nominal definition aims to provide a reliable means of selecting genuine instances of the investigated phenomena.⁶ Adopting the same attitude, Sorabji maintains that a nominal definition (which he also identifies with the answer to the question of "whether it is") determines existence, when it can be applied to concrete instances.⁷

⁴ In the following I reject this interpretation. My alternative view concerning this query is discussed in section 5.3.

⁵ Bolton, "Essentialism and Semantic Theory in Aristotle's *Posterior Analytics* II. 7-10" (1976), p. 529.

⁶ Demoss and Devereux, "Essence, Existence and Nominal Definition in Aristotle's *Posterior Analytics*" (1988), p. 143.

⁷ Sorabji, "Definitions: Why Necessary and in What way?" (1981), pp. 221-222. For the untenability of identification of the definition that answers the question *ei esti* with nominal definitions, see my discussion of Aristotle's employment of the expression *ti sêmainei to onoma* in the *Posterior Analytics* II.7. David Charles's interpretation, like my interpretation, keeps distinct the questions *ti sêmainei to onoma* and *ei esti*. In Charles's view Aristotle separates three distinct stages of demonstrative enquiry: (1) an account of what a name signifies; (2) an account of whether the object signified by a name exists; (3) an account of the essence of the object signified by a name. [Charles, *Aristotle on Meaning and Essence* (2000), pp. 24-56]. Although I agree with Charles that the

Such an interpretation of the relation between definition and being does not accord with Aristotle's position on the question of "whether it is". The above quoted passage from the *Posterior Analytics* II.8 implies that nothing but the verbal account is required for determining being. The existential interpretation, by contrast, explains the relation between the nominal definition and being as an application of a general description to a particular instance, which becomes familiar through perception. According to the existential interpretation, it is not the verbal account in itself that determines being; the correspondence between the verbal account and our experience does so. A verbal account does not determine being unless it can be applied to certain particular instances. In this case the conviction that a certain thing exists stems from a perceptual acquaintance with particular instances and not from the verbal account. In the *Posterior Analytics* II.8 Aristotle does not mention acquaintance with particular instances as included in the answer to the question of "whether it is", but elsewhere he considers such an acquaintance inadequate for determining being. In *Metaphysics* VII.10 he says that particular objects cannot be defined since "when they go out of our actual consciousness, it is not clear whether they exist or not" (1036a5-7). That is, a perceptual acquaintance with a particular instance cannot be regarded as yielding knowledge of being since perceptual acquaintance cannot determine whether this instance exists beyond our consciousness. Accordingly, the relation between a verbal account and being needs to be interpreted differently.

The idea that a verbal account determines being is not foreign to Aristotle's thought. For instance, throughout his *Physics*, questions such as whether void or time exist are answered by verbal characterizations of them.⁸ In their analyses of the uses of the verb "to be" in Greek philosophy, Owen and Kahn have argued, albeit on different grounds, that the notion of existence is almost absent from Greek thought. The verb "to be" does not denote existence in the sense of "there is an X" but is employed in a predicative sense, i.e., "X is Y". In this sense being means "to be something"; to have certain characteristics.⁹ Still, considering the question of "whether it is" in the *Posterior Analytics*, both Owen and Kahn argue that the verb "to be" has an existential meaning.¹⁰ In the following discussion of

questions of signification and existence are distinct, I do not ascribe to accounts of what names signify a specific role in demonstrative enquiry.

⁸ *Physics* IV.7, 10.

⁹ Owen, "Aristotle and the Snares of Ontology" (1965), p. 76; Kahn, "Retrospect on the Verb 'To Be' and the Concept of Being" (1986), pp. 21-22.

¹⁰ Kahn, "Why Existence does not Emerge as a Distinct Concept in Greek Philosophy" (1976), pp. 326; Owen, "Aristotle and the Snares of Ontology" (1965), p. 84.

how definitions determine being, I show that the question of “whether it is” does not carry existential implications.

In the *Posterior Analytics* II.8 Aristotle draws a distinction between accidental and non-accidental grasp of being. This distinction serves as a criterion for distinguishing verbal accounts that determine being from those that do not.

Now in cases in which we know accidentally that a thing is, necessarily we have no hold on what it is; for we do not even know that it is, and to seek what it is without grasping that it is, is to seek nothing. (93a24-27)

In the *Posterior Analytics* II.7 Aristotle draws a distinction between the question of “what it is” (*ti esti*) and the question of “what a name signifies” (*ti sêmeinei to onoma*). As this distinction is introduced in distinguishing between non-existent and existent entities, it seems to correspond to the distinction between the accidental and the non-accidental grasp of being.

For it is necessary for anyone who knows what a man or anything else is to know too that it is (for of that which is not, no one knows what it is – you may know what the account or the name signifies when I say goatstag, but it is impossible to know what a goatstag is). (92b4-8)

According to this passage, the distinction between a real definition (*ti esti*) and a nominal definition (*ti sêmeinei to onoma*) corresponds to the distinction between real and non-existent objects. Non-existent objects are characterized as definable only in the nominal sense whereas existent objects are marked as those and only those objects to which an essence (*ti esti*) can be ascribed. The possibility of answering the question of “what it is” serves as a criterion for distinguishing existent objects from non-existent objects. Therefore, a genuine answer to the question of “what it is” has to be answered by a definition that points to the essence. Further analysis of Aristotle’s distinction between definitions and accidental accounts reinforces this conclusion.

In his discussion of definition in *Metaphysics* VII.12 Aristotle compares the relations subsisting between the components of definitions with those subsisting in accidental accounts. He exemplifies definitions and accidental accounts respectively by the compound expressions: “a biped animal” and “a white man”. Analyzing the difference between them, Aristotle maintains that the compound expression “a biped animal” differs from an accidental compound in conveying one meaning, i.e., the meaning of “man”. The accidental expression “a white man” has no unitary meaning but expresses two separate meanings, namely those conveyed by the words “man” and

“white” (1037b9-20). Further, Aristotle asserts that even if accidental expressions are denoted by a single word they do not form a unitary meaning. In *On Interpretation* VIII he considers the case in which a single word, such as “cloak”, is assigned to two different concepts, e.g., “man” and “horse”. In this case, Aristotle contends, the proposition “the cloak is white” cannot be regarded as a single affirmation but should be analyzed to the propositions “the man is white” and “the horse is white” (18a18-21). Hence, an essential account differs from an accidental account in forming one single meaning. This criterion can be applied to the distinction made in the *Posterior Analytics* II.7 between nominal definitions and real definitions.

...If a definition has nothing at all to do with what a thing is, it will be an account signifying the same as a name. But that is absurd. For, first, there would be definitions even of non-substances, and of things that are not – for one can signify even things that are not. Again, all accounts will be definitions; for one could posit a name for any account whatever, so that we would all talk definitions and the *Iliad* would be a definition. (92b27-32)

In the *Posterior Analytics* II.7 Aristotle presents the terms “goatstag” (92b7) and the “*Iliad*” (92b32) in exemplifying entities, to which the question of “what the name signifies” applies. Both examples illustrate a case in which a single name does not convey one single meaning. The *Iliad* is the name of the Homeric epic, i.e., Homer’s tales of the Trojan War constitute the meaning of “*Iliad*”. Yet the word “*Iliad*” expresses not just one single meaning but a complex of meanings, which correspond to the various scenes and stories depicted in the *Iliad*.¹¹ Likewise, the word “goatstag” is a single word that does not denote one single meaning but refers to two different animals, i.e., a goat and a stag. So a nominal definition, which is the only definition that can be given to non-substances and non-existent entities, does not constitute one single meaning.

Consequently, the distinction in *Posterior Analytics* II.8, between accidental and non-accidental answers to the question of “whether it is” is equivalent to Aristotle’s distinction between the question of “what it is” and the question of “what a name signifies”. It follows that the question of “whether it is” is to be answered by a real definition, while a nominal definition marks non-existent entities. The correspondence between nominal

¹¹ In the *Poetics* XX Aristotle expresses this view explicitly: “A sentence is said to be one in two ways, either as signifying one thing or as a union of several speeches made into one by conjunction. Thus the *Iliad* is one speech by conjunction of several; and the definition of man is one through its signifying one thing” (1457a27-30).

definitions and non-existent entities indicates that Aristotle presupposes the relation between real definitions and being at the outset. Existent and non-existent entities are distinguished in terms of two types of verbal accounts: a definition, which forms a single meaning, and name's signification (*ti sêmeinei to onoma*), which does not. Further, Aristotle's notion of a real definition intrinsically includes the notion of being, since it is formed, at the outset, in opposition to a verbal account, which refers to the linguistic realm, i.e., to names.

In light of the equivalence between Aristotle's notions of definition and being, the question of the relation between definition and being may be answered. In *Metaphysics* VII.4 Aristotle distinguishes a definition from a name's signification in answering the question of what a substance is.

But we have a definition not where we have a word and a formula identical in meaning (for in that case all formulae would be definitions; for there will be some name for formula whatever, so that even the *Iliad* would be a definition), but where there is a formula of something primary; and primary things are those which do not involve one thing's being said of another ... but for everything else as well, if it has a name, there will be a formula of its meaning – viz. that this attribute belongs to this subject. (1030a7-16)

Definition, then, unlike the signification of a name, determines being since it can be given only to primary things, i.e., to substances. Such a definition does not determine being in the existential sense; it does not assert, "there is an X". The expression "there is an X" refers to particular instances¹² while according to Aristotle, a particular instance cannot be defined (e.g., *Metaphysics* 1039b27-31). Rather, a definition distinguishes an object from all other objects, and thereby indicates that the defined object is a certain (*tode ti*) unitary entity. Definition determines being in the sense of being "this" (*tode ti*). Although in the *Categories* Aristotle mentions particulars as examples for "thisness" (3b10-18), in *Metaphysics* VII he employs the expression *tode ti* in reference to forms (1037b27) or to composites of matter and form (1033a31; 1033b19-26; 1037a1-2; 1038b5-6). In *Metaphysics* VII.3 Aristotle rejects the contention that matter is the substance, claiming that matter is not "a this" (1029a28), while in *Metaphysics* VIII.1, he contends that matter is merely potentially "a this" (1042a26-28). It may be

¹² The plural expressing "there are such things as x's" also refer to particular instances, taken as forming a collection of items.

concluded first that “thisness” is determined by the form,¹³ and second, that the composite of matter and form is a “this” due to its universal aspect, i.e., the form.¹⁴

A comparison between Aristotle’s notion of being and the modern notion of existence clarifies further the equivalence between definition and being. Since the Middle Ages essence and existence have been regarded as polar concepts. This conception finds expression in the modern distinction between concept and object, which corresponds to the distinction between nominal definition and real definition. Generally, a nominal definition is regarded as referring to a concept by specifying the general characteristics of a term’s meaning. Such a definition involves no assertion regarding the existence of the defined object. A real definition examines the conditions under which such a concept is realized. A real definition therefore refers to characteristics taken from experience, in order to determine whether the existence of the defined object is compatible with other existent objects.¹⁵ An application of this account of the distinction between nominal and real definitions to Aristotle’s distinction between signification and essence leads to a certain confusion. From an Aristotelian point of view the above notion of nominal definition appeals to the essential features of an object, whereas a real definition appeals to its accidental aspects. The above notion of nominal definition corresponds to Aristotle’s notion of real definition, since from Aristotle’s point of view a definition that specifies the essence can be given to existent things alone. Further, the above notion of real definition corresponds to Aristotle’s notion of nominal definition since a definition that lists the accidental attributes, taken from experience, answers in Aristotle’s view the question of “what a name signifies” (*ti sêmeinei to onoma*). Accordingly, Aristotle’s notion of being and the modern notion of existence

¹³ For a different view see Gill, *Aristotle on Substance: The Paradox of Unity* (1989), pp. 86-90.

¹⁴ Aristotle’s contention that a form is *tode ti*, has been interpreted by some commentators as implying particular forms, which are proper to a single entity [Sellars, “Substance and Form in Aristotle” (1957), pp. 688-699; Lloyd, *Form and Universal in Aristotle* (1981), p. 24; Annas, *Aristotle’s Metaphysics: Books M & N* (1976) 190-191; Frede and Patzig, *Aristoteles, Metaphysik Z: Text, Übersetzung und Kommentar* (1988), pp. I. 48-57]. However, many of Aristotle’s passages indicate that the form is common to a number of individuals (e.g., 1034a5-8). For this view see Lear, “Active Episteme” (1987); Woods, “Universals and Particular Forms in Aristotle’s *Metaphysics*” (1991); Lewis, *Substance and Predication in Aristotle* (1991), pp. 151-158 and 308-336.

¹⁵ One of the conspicuous expressions of the distinction between essence and being is to be found in the ontological argument for the existence of God. The very need to prove God’s existence from the definition of God implies that the relation between essence and being is not self-evident but has to be demonstrated.

are radically different; Aristotle's belongs to universal concepts, whereas modern thought assigns existence to particular objects alone.

Preoccupied with the modern notion of existence, Aristotle's modern commentators interpret his distinction between the questions "what it signifies" and "what it is" in terms of the distinction between nominal and real definitions. Therefore, they appeal to experience in construing the relation between definition and being. But as I have shown, the distinction between nominal and real definition cannot be applied to Aristotle's discussion, since the Aristotelian counterpart of nominal definitions, i.e., the answer to the question of "what a name signifies" is not regarded by him as a definition at all, while from an Aristotelian point of view the modern notion of nominal definition expresses reality.

This analysis of Aristotle's notion of "being" carries an implication for his notion of demonstration. The equivalence between being and definition seems to indicate that from Aristotle's standpoint, proof and explanation coincide. Proving a fact and explaining it are conceived of as one and the same procedure. Before elaborating on this, I account in the following section for the distinction between the questions of "whether it is" and "what it is".

5.3 BEING AND ESSENCE

In the *Posterior Analytics* II.2 Aristotle characterizes the demonstrative procedure as a means of turning knowledge of the fact and knowledge of being into knowledge of the reason why and knowledge of the essence (*ti esti*) (89b36-90a6). Since a genuine answer to the question of "whether it is" involves a certain acquaintance with the essence (93a21-24), it might be asked in what respect do knowledge of the fact and knowledge of being differ from full-fledged knowledge, which is attained by means of demonstrative syllogism.

Aristotle's examples of the answers to the question of "whether it is" indicate that an account of being is a vague definition; thunder is a certain (*tis*) noise in the clouds, or eclipse is a certain (*tis*) privation of light (93a22-23). These examples imply that an account of being would result in an account of the essence and of the reason why, if the indefinite pronoun "*tis*" is replaced by a specific account. However, Aristotle's more elaborate examples of knowledge of the fact suggest that substituting the indefinite pronoun "*tis*" by a more definite account is insufficient to turn these vague definitions into accounts of the essence. In the *Posterior Analytics* II.8

Aristotle presents two syllogisms, having the proposition “eclipse holds for the moon” as their conclusion:

- (I) (1) Eclipse holds for screening by the earth.
 (2) Screening by the earth holds for the moon.
 (3) Eclipse holds for the moon (93a29-31).

- (II) (1) Eclipse holds for the inability to produce a shadow during a full moon, although there is nothing evident between us and the moon.
 (2) The inability to produce a shadow during a full moon, although there is nothing evident between us and the moon, holds for the moon.
 (3) Eclipse holds for the moon (93a36-b3).

According to Aristotle, syllogism (I) yields knowledge of the reason why as well as knowledge of the fact, while syllogism (II) yields knowledge of the fact alone. Considering the different middle terms of these two syllogisms, it seems that vagueness is not the feature that distinguishes them. The middle term of syllogism (I) is no more definite than that of syllogism (II). Rather, these two middle terms appear to differ qualitatively. The middle term of the second syllogism accounts for an eclipse on the basis of a perceptual acquaintance with the phenomenon, that is, by referring to the way in which this phenomenon is seen (the lack of a shadow). By contrast, the middle term of the first syllogism specifies a situation that cannot be seen from earth. Knowledge of the fact apparently differs from knowledge of the reason why in that the former is anchored in perception.

This conclusion is reinforced by Aristotle’s discussion of knowledge of the fact *versus* knowledge of the reason why in the *Posterior Analytics* I.13. This distinction is commonly understood as one between non-explanatory and explanatory deductions. However, an exhaustive interpretation of Aristotle’s distinction has to answer the question of what Aristotle considers an explanation; it should disclose his criterion for distinguishing explanatory from non-explanatory deductions. The *Posterior Analytics* I.13 opens with a distinction between two cases, in which demonstration yields knowledge of the fact alone: (1) demonstration that is not based on the immediate primitive cause; (2) demonstration that is based on an immediate term, but the less familiar of two converting terms. In characterizing the second type of demonstration of the fact, Aristotle introduces a criterion that distinguishes an explanatory from a non-explanatory middle term: knowledge of the fact is attained if the demonstration does not proceed from the more familiar of two terms. In the *Posterior Analytics* I.2 Aristotle maintains that a term may be

more familiar in two ways: "I call prior and more familiar in relation to us what is nearer to perception, prior and more familiar *simpliciter* what is further away" (72a1-4). Although the *Posterior Analytics* I.13 does not explicate the exact sense of "more familiar" that is employed in this context, Aristotle's evaluation that demonstration of the fact as deficient in a way indicates that demonstrations of the fact are based on middle terms that are more familiar in relation to us. Further, Aristotle's examples support the view that demonstration of the fact is anchored in perception. Exemplifying the second type of demonstration of the fact, Aristotle introduces two astronomical syllogisms; in the first, non-scintillation explains the position of the planets (78a30-34) while in the second the spherical shape of the moon is explained by its waxing (*auxêsis*) (78b4-13). As Barnes points out, the modern notion of causality and explanation cannot account for the priority of the explanatory middle terms (i.e., the position of the planets or the spherical shape of the moon) over the non-explanatory middle terms. According to the modern notion of explanation, both demonstrations would be considered explanatory.¹⁶ However, Aristotle's distinction between the two senses of "more familiar" clarifies the difference between these examples. The non-explanatory middle terms, i.e., non-scintillation and the increase in the extent of the illuminated portion of the moon before the full moon, are both visual phenomena. By contrast, the position of the planets and the spherical shape of the moon are derived from theoretical considerations; both are obtained by geometrical considerations.

Furthermore, Aristotle employs the distinction between knowledge of the fact and knowledge of the reason why in classifying the various branches of knowledge. Some branches of knowledge are subordinate to others since the latter demonstrate the reasons for facts that have been established by the former. For instance, geometry proves the reasons for the facts proved by optics; solid geometry and mechanics are similarly related, as are arithmetic and harmonics (78b23-79a7). Aristotle regards optics, mechanics, and harmonics as the more natural branches of mathematics.¹⁷ Geometry, for instance, investigates natural lines irrespective of their perceptual properties, while optics investigates geometrical lines with regard to their perceptual properties. Likewise, mechanics and harmonics investigate perceptual properties by applying mathematical considerations to them. Consequently, knowledge of the fact and knowledge of the reason why differ qualitatively; the former is based on perceptual acquaintance while the latter is conceptual.

Moreover, such an interpretation of the distinction between knowledge of the fact and knowledge of the reason why can explain the priority of the

¹⁶ Barnes, *Aristotle's Posterior Analytics* (1993), p. 156.

¹⁷ *Physics* II.2, 194a10; *Metaphysics* II.2, 997b15-21; XIII.3, 1078a14-18.

question of “whether it is” over the question of “what it is”. Throughout the *Posterior Analytics* and in other contexts Aristotle holds that knowledge of the fact ought to precede the answer to the question of “what it is”. However, the reverse order of questions seems to be more plausible; for one has to know what one is looking for in order to look for it. If the distinction between the “is” and the “what” is understood to correspond to the distinction between perception and thought, the priority of the “is” over the “what” can be explained in terms of Aristotle’s distinction between knowledge of the fact and knowledge of the reason why. The above analysis of the *Posterior Analytics* I.13 shows that the distinction between knowledge of the fact and knowledge of the reason why corresponds to the distinction between perception (i.e., things that are more familiar to us) and conceptualization (i.e., things that are more familiar by nature). Hence the priority of knowledge of the fact over knowledge of the reason why can be explained on the basis of the chronological priority of perception over thought.¹⁸

Knowledge of the fact has been interpreted here as anchored in perception, while in the foregoing discussion of the relation between signification and being the possibility of determining being through experience was rejected. The difference between the perceptual facet of knowledge of the fact and experience is clarified in the following passage from the *Topics* VI.4:

Absolutely, then, it is better to try to come to know what is posterior through what is prior, inasmuch as such a way of procedure is more scientific. Of course, in dealing with persons who cannot recognize things through terms of that kind, it may perhaps be necessary to frame the account through terms that are familiar to them. Among definitions of this kind are those of a point, a line, and a plane, all of which explain the prior through the posterior; for they say that a point is the limit of a line, and a line of a plane, a plane of a solid. (141b15-22)

¹⁸ Faced with the problem of the priority of the question of “whether it is” over the question of “what it is”, commentators appeal to the explanatory import of the demonstrative syllogism. Ackrill, for instance, claims that the question of the fact precedes the question of the reason why, since explanation presupposes the explained fact [Ackrill, “Aristotle’s Theory of Definition: Some Questions on *Posterior Analytics* II. 8-10” (1981), p. 365]. This interpretation may account for Aristotle’s contention insofar as it is stated in demonstrative contexts. However, Aristotle’s contention that the question of “whether it is” precedes the question of “what it is” appears in non-demonstrative contexts as well. The interpretation presented above offers a general account of this contention.

The definitions described in this passage characterize the terms point, line, and plane by appealing to perception¹⁹ but these terms do not denote perceptual objects. Although one cannot have an empiric experience of geometrical objects one may define them by appealing to perceptual considerations. In the *Physics* II.2 Aristotle claims that the mathematician does not treat geometrical objects as if they were natural objects (193b31-32). As he glosses to this, he adds that the treatment of geometrical objects as if they were natural objects involves regarding them as limits of natural bodies. Thus, the geometrical definition, which is presented in the passage above, defines a non-empirical object by taking into account perceptual considerations. Knowledge of the fact, I argue, is attained by a definition of this type.

Being expressed in a definition, knowledge of the fact cannot be equated with experience; it is rather a perceptual understanding. Perceptual understanding is primarily a sort of understanding; it differs from experience in having a universal validly; unlike experience, perceptual understanding grasps the typical features of particular instances. Aristotle's contention, in the *Posterior Analytics* II.2, that a perception of a moon eclipse from the moon leads to knowledge of the universal (90a26-29) exemplifies such a perceptual understanding. In the case described by Aristotle, one attains a universal understanding of a certain phenomenon, although one only perceives a particular instance. Further, as I have shown in the first chapter, Aristotle employs the term *aisthesis* to convey this meaning; in some contexts he considers perception an apprehension of the universal.

In contrast to knowledge of the fact, which is a perceptual understanding, knowledge of the reason why, or an answer to the question of "what it is" involves a conceptual understanding. In the *Topics* VI.4 Aristotle contrasts the definition of geometrical objects discussed above with a definition that defines the object according to its genus and *differentiae* (141b25-28). He conceives of such a definition, which subsumes an object under its generic class, as exhibiting the real order of being. A definition according to genus and *differentiae* is more familiar by nature, in being detached from perception. Such a definition goes beyond perceptual consideration, in capturing not only the universal aspect of the object but its essential, i.e., conceptual, facet as well.

Accordingly, knowledge of the fact and knowledge of the reason why are two types of understanding – perceptual and conceptual. Both types capture

¹⁹ In this chapter from the *Topics* Aristotle regards geometrical definitions that define the lower dimensions in terms of the higher dimensions as based on perception. At lines 141a9-11 he says: "for a solid falls under perception most of all and the plane more than a line and a line more than a point". Thus a definition that defines a point in terms of lines, or lines in terms of planes appeals to perceptual considerations.

the universal facet of the object, yet each articulates this universal facet differently. In answering the question of “whether it is”, one attains an understanding of the object. But this is not considered full-fledged knowledge since perceptual understanding does not capture the essence of the object, according to its conceptual characterizations. Since it is anchored in perception, such an understanding views the object from a human perspective and hinders the full apprehension of its real essence, i.e., the essence that is independent of human attitudes. By contrast, conceptual understanding is based on an apprehension of a phenomenon in light of Aristotle’s primary ontological entities. He does not conceive of genera and species as creations of the human mind. From an epistemological perspective they are regarded as things to be revealed not created. Therefore, Aristotle regards conceptual understanding as full-fledged knowledge as it is dependent on reality itself.

The demonstrative procedure, which proceeds from an answer to the question of “whether it is” and ends with an answer to the question of “what it is”, is a mediator between perceptual and conceptual understanding. Therefore, syllogistic reasoning is the only adequate means of attaining knowledge in the Aristotelian sense. As I have shown, syllogistic logic is a logic of understanding, while syllogistic deduction is a process, in which one content is derived from another. Since Aristotle conceives of knowledge as conceptual understanding that follows from perceptual understanding, a deductive procedure that makes possible a modification of understanding is considered indispensable for attaining knowledge. In the following section I offer an analysis of the modification that perceptual understanding undergoes through the demonstrative procedure.

5.4 DEMONSTRATION AND ESSENCE

The *Posterior Analytics* II.3-10 deals with the question of whether definitions can be demonstrated; that is, whether definitions can serve as conclusions in demonstrative syllogisms. In concluding the dialectical discussion of the relation between demonstration and definition in the *Posterior Analytics* II.8, Aristotle states his own view:

Since, as we have said, to know what something is and to know the cause of the fact that it is are the same – the argument for this is that there is some cause, and it is either the same thing or something else, and if it is something else it is either demonstrable or non-demonstrable – if, then, it is something else and it is possible to demonstrate it, it is necessary for the

cause to be the middle term and to be proved in the first figure;
for what is being proved is both universal and affirmative.
(93a3-9)

Since antiquity, demonstration of the essence has been interpreted as a demonstration that has as its conclusion a partial definition, called material definition (*horismos hulikos*), and includes in its first premiss another partial definition, called formal definition (*horismos eidikos*). Philoponus, for instance, views the following as a demonstration of the essence:

- (1) Anger is a desire for revenge;
- (2) A desire for revenge produces boiling of the blood about the heart;

Therefore (3) anger is boiling of the blood about the heart.²⁰

This demonstration includes a formal definition in the major premiss and a material definition in the conclusion. Both definitions are partial while the full definition is obtained by the combination of these. For instance, the full definition of anger is “a boiling of the blood about the heart, caused by the desire for revenge”. Philoponus’ interpretation has been introduced in order to accommodate Aristotle’s contention that the essence can be demonstrated with his claim that a demonstration having a definition as its conclusion is circular. According to this interpretation a definition can be demonstrated insofar as the demonstration is not held to deduce a full definition. The demonstration introduced above deduces a partial definition, while the full definition is exhibited indirectly through the demonstration. This interpretation is restated in many recent interpretations of the second book of the *Posterior Analytics*.²¹

However, if we accept the validity of demonstrations that have a partial definition as their conclusion, Aristotle becomes exposed to his own criticism.²² In the *Posterior Analytics* II.3 he distinguishes a definition and a demonstrative conclusion:

²⁰ Philoponus, in *Aristotelis Analytica Posteriora Commentaria.*, p. 365.27-8.

²¹ Le Blond, *Logique et méthode chez Aristote* (1970), pp. 158-59; Mansion, *Le jugement d'existence chez Aristote* (1976), pp. 191-92; Barnes, *Aristotle's Posterior Analytics* (1993), p. 217; Bolton, “Essentialism and Semantic Theory in Aristotle’s *Posterior Analytics* II. 7-10” (1976), p. 521; Demos and Devereux, “Essence, Existence and Nominal Definition in Aristotle’s *Posterior Analytics*” (1988) p. 136. Ackrill [“Aristotle’s Theory of Definition: Some Questions on *Posterior Analytics* II. 8-10” (1981) p. 361] presents a modified version of this view, claiming that the full definition is not demonstrated but is revealed through the demonstration, and that the fact (*to hoti*) is in a sense a partial definition.

²² An elaborate criticism of the traditional view is presented in Landor, “Aristotle on Demonstrating Essence” (1985). See also Charles, *Aristotle on Meaning and Essence* (2000), pp. 64-67.

Again, every demonstration proves something of something, i.e. that it is or it is not; but in a definition one thing is not predicated of another – e.g. neither animal of two-footed nor this on animal, nor indeed figure of plane ... (90b33-37).

Here Aristotle argues that a demonstrative conclusion expresses a predicative relation whereas a definition does not involve predicating one thing of another. Since Aristotle views definitions as unitary accounts, the distinction between definitions and conclusions can be construed as one between predications in which subject and predicate have equivalent meanings, and predications in which subject and predicate connote different meanings. This interpretation is in line with Aristotle's contention that the genus and the *differentiae* are two aspects of the same thing (1045a23-25). The conclusion of a demonstration, then, is a proposition in which subject and predicate express two different meanings, while viewing the demonstrative conclusion as a partial definition eradicates Aristotle's distinction between definition and predication.

Moreover, the traditional interpretation fails to portray the explanatory relations that subsist between the premisses and the conclusion. Since they claim that the demonstrative conclusion is a partial definition, the proponents of this view have to conceive of the middle term as the cause or explanation of the partial definition. According to this interpretation, the middle term has to explain why a certain definition holds for the defined term, for instance, why thunder is a noise in the clouds. But in the *Posterior Analytics* II.7 Aristotle claims that no demonstration can clarify why one term defines another (92b19-25). He implies that it is not possible to explain why a definition is correct. If we endorsed the traditional interpretation, maintaining the explanatory role of the middle term, we would be averring that the formal definition could explain the material definition. This conclusion does not agree with Aristotle's contention that definitions cannot be explained.

By contrast, Aristotle's own view of the relation between demonstration and definition suggests that a demonstration of the essence is not tantamount to a demonstration having a definition as its conclusion. In the *Posterior Analytics* II.8 he exemplifies demonstration of the essence by the following syllogisms:

- (1) Eclipse holds for screening by the earth.
- (2) Screening by the earth holds for the moon.
- Therefore (3) Eclipse holds for the moon.
- (1) Thunder holds for extinction of fire.
- (2) Extinction of fire holds for the cloud.

Therefore (3) thunder holds for the cloud.

The conclusions of these syllogisms are not partial definitions, but statements of facts. In the *Posterior Analytics* II.8 Aristotle conceives of these syllogisms as a means by which the essence is revealed:

...hence no deduction and no demonstration of what a thing is comes about – yet it is clear through deduction and through demonstration. Hence without a demonstration you cannot become aware of what a thing is ... yet there is no demonstration of it ... (93b16-19)

The relation between the above demonstrations and the essence can be explained in light of the explanatory role of the middle term. In the *Posterior Analytics* II.2 Aristotle identifies the question of “why it is” with the question of “what it is” (90a14-15). He then contends that the middle term of a demonstration is the cause of the attribute of the conclusion (90a10-11). Applying these contentions to Aristotle’s view of the relation between demonstration and essence, it may be concluded that the middle term is the essence that is revealed by demonstration. One can derive definitions from demonstrations by predicating the middle term of the major term. For instance, eclipse is defined as screening by the earth and thunder as extinction of fire.

In his recapitulation of the discussion of definitions and demonstrations in the *Posterior Analytics* II.10 Aristotle lists three types of definitions: (1) an indemonstrable account of what a thing is; (2) a deduction of the essence, differing in aspect from demonstration; (3) the conclusion of a demonstration of the essence. The last two types of definitions seem to correspond to the two senses of demonstration of the essence, which were introduced in the *Posterior Analytics* II.8. The conclusion of a demonstration of the essence corresponds to a demonstration in which one definition is deduced from another, since such a demonstration has a definition as its conclusion. Aristotle considers this type of definition *logikos syllogismos*, that is, not as a demonstration in the strict sense. While he regards the conclusion of a demonstration of the essence as a definition, he does not consider the deductive procedure leading to this definition a genuine demonstration.

The second type of definition (i.e., a deduction of the essence, differing in aspect from demonstration) indirectly reveals the essence by revising the order of the syllogistic terms. This type of definition corresponds to the eclipse and thunder demonstrations discussed above. In the *Posterior Analytics* II.8 Aristotle analyzes the relationship between demonstration and essence in terms of the explanatory import of the middle term. Likewise,

introducing the second type of definition in the *Posterior Analytics* II.10, Aristotle distinguishes demonstration and definition by appealing to the explanatory import of demonstrations:

For there is a difference between saying why it thunders and what thunder is; for in the one case you will say: Because the fire is extinguished in the clouds. What is a thunder? – A noise of fire being extinguished in the clouds. Hence the same account is put in a different way, and in this way it is a continuous demonstration, in this way a definition. (94a3-7)

This passage indicates that a demonstration that reveals the essence indirectly is regarded as a definition only in the sense that the content expressed in a definition appears in such a demonstration in a revised way. This type is not a definition in the strict sense since it is not formulated as such. Consequently, the second and third senses of definitions differ: the second sense (i.e., a deduction of the essence, differing in aspect from demonstration) is a demonstration in the strict sense, and a definition in a weaker sense. By contrast, the third sense of definition (i.e., the conclusion of a demonstration of the essence) is a definition in the strict sense but a demonstration in a loose sense.

The traditional interpretation of demonstration of the essence, which views it as a demonstration that concludes a partial definition, raises an interpretative problem that otherwise could be avoided. This interpretation confines demonstration and demonstrative knowledge to definitions, while Aristotle exemplifies demonstrative conclusions by propositions, such as “the moon is eclipsed”, which express facts and not definitions. Acknowledging the traditional interpretation, commentators attempt to accommodate Aristotle’s contention that demonstration proves the *per se* incidentals²³ with this narrow interpretation of the demonstrative procedure.²⁴

In light of the foregoing analysis it seems inadequate to impute such a narrow view of demonstrative knowledge to Aristotle. A demonstration reveals the essence without concluding a definition. In the *Posterior Analytics* I.10 Aristotle distinguishes two prior assumptions that are required for forming a demonstration. The being (*hoti esti*) and the signification (*ti sêmainei*) of some things, called primitives (*ta prôta*) have to be assumed, while of other things, called dependents (*ta ek toutôn*), only the signification

²³ According to *Metaphysics* V.30, *per se* incidentals are attributes that belong to a subject in itself, but not as part of its substance.

²⁴ This task has been undertaken by McKirahan, *Principles and Proofs – Aristotle’s Theory of Demonstrative Science* (1992); Ferejohn, *The Origins of Aristotelian Science* (1991); Goldin, *Explaining an Eclipse* (1996).

has to be assumed and their being has yet to be proved (76a32-34).²⁵ Aristotle's classification of the ingredients of a demonstrative branch of knowledge indicates that the distinction between primitive and dependent terms corresponds with the grammatical distinction between the subject and the predicate of the conclusion (76b11-12). According to the *Posterior Analytics* I.10, the role of demonstration is to prove the being of the predicate of the conclusion.

Prior to the demonstration, the prover posits the meaning of the conclusion's predicate. Nevertheless, the proposition that states the predicate's meaning is not the conclusion of the demonstration. A demonstration that reveals the essence proceeds from a definition of the predicates of the conclusion, such as "eclipse" and "thunder". This definition answers the question of "whether it is"; it specifies the perceptual understanding of these terms. Nevertheless, the explicit answer to the question of "whether it is" does not constitute the conclusion of a demonstration. The demonstrative conclusion is a predicative proposition, such as "the moon is eclipsed", in which the predicate, e.g., "eclipse", implicitly carries the very meaning that is expressed by the question of "whether it is". In other words, prior to the demonstration, the predicate of the conclusion carries a meaning that arises from its perceptual understanding. For instance, before the demonstration the term "eclipse" means "an inability to produce a shadow during a full moon although there is nothing evident between us and the moon". A full-fledged demonstration exhibits the essence of the major term (i.e., the predicate of the conclusion) through the syllogistic middle term. After the demonstration the predicate of the conclusion bears its conceptual meaning; for example, after the demonstration "eclipse" means "screening by the earth". By means of demonstrations the meaning of the major term undergoes a modification; it changes from a perceptual understanding into a conceptual understanding.

Having characterized the stages before and after the demonstration, we may now consider Aristotle's notion of demonstration. The most conspicuous difference between this and the modern notion of proof may be described as follows. Modern thought conceives of a proof as leading from premisses to conclusions, while Aristotle's demonstration seems to lead from conclusions to premisses. In the *Posterior Analytics* I.22 Aristotle describes the demonstrative procedure: "it is by interpolating a term inside

²⁵ The distinction between primitive and dependent terms has been interpreted in light of modern axiomatics as identical with the distinction between undefined terms and derived terms [Scholz, "The Ancient Axiomatic Theory" (1975), p. 60]. However, since Aristotle explicitly states that both types of terms should be defined in advance, applying the modern distinction between undefined and defined terms discerns primitive and dependent terms by the very feature in which, according to Aristotle, they do not differ.

and not by taking an additional one that what is demonstrated is demonstrated" (84a36-37). Demonstration is characterized as mediating between two terms. Aristotle conceives of the predicative relations between the subject and the predicate of the conclusion as relations that are known in advance, while the demonstrative procedure aims to find the middle term, which mediates between these two terms.²⁶

Aristotle's notion of demonstration differs in another respect from the modern notion of proof. When analyzing the ingredients of a demonstrative branch of knowledge, Aristotle does not consider the conclusion as a whole, but examines its subject and predicate separately. His discussion in the *Posterior Analytics* I.10 indicates that although both the subject and the predicate of the conclusion are known in advance they are not equally known prior to the demonstration. According to Aristotle, a demonstration proceeds from a full-fledged knowledge of the conclusion's subject; prior to the demonstration, its being as well as its meaning are assumed, whereas the knowledge of the predicate is partial since only its meaning is assumed in advance. Aristotle's distinction between the subject and the attribute of the conclusion implies that demonstrations aim to establish the conclusion's predicate. In the *Posterior Analytics* II.9 Aristotle indeed distinguishes primitive terms, which include their cause within themselves from terms that do not. This distinction corresponds to the distinction between a substance, which can serve solely as a subject of predication, and an attribute, which is the predicate of a proposition. In the *Posterior Analytics* II.9 Aristotle confines demonstrations to terms whose cause is different from themselves; the object of demonstration is the predicate of the conclusion.

Bearing in mind Aristotle's attitude to logical entailment and the contention that the object of demonstration is the predicate of the conclusion, we are now in a position to determine the role of demonstrative syllogism. In the *Posterior Analytics* I.7 and I.10 Aristotle offers different accounts for the role of demonstration. In chapter I.7 he says:

For there are three things in demonstration: one, what is being demonstrated, the conclusion (this is what belongs to some genus in itself); one, the axioms (axioms are the things on which the demonstration depends); third, the underlying genus of which the demonstration *makes clear* (*dêloi*) the attributes and what is accidental in itself. (75a38-b2)²⁷

²⁶ For further support of the view that knowledge is an inquiry into first principles, rather than a derivation from first principles, see Wieland, "Aristotle's *Physics* and the Problem of the Inquiry into First Principles" (1975).

²⁷ My emphasis.

By contrast, in the *Posterior Analytics* I.10 Aristotle claims that demonstration proves the attribute's being (76a34). The apparent discrepancy between these two passages is resolved in light of the foregoing discussion. An Aristotelian demonstration, unlike modern proofs, proceeds from the conclusion to the premisses; a demonstration is a search for the appropriate middle term. Accordingly, a demonstration results in subsuming the attribute of the conclusion under a general term. For instance, through demonstration the term "eclipse" is subsumed under the general term "screening by the earth". Demonstration aims to define or clarify the attribute of the conclusion.

Nevertheless, it would be inadequate to construe Aristotle's notion of demonstration as merely a deductive explanation. The preceding analysis of the relation between definition and being suggests that Aristotle views definition as implying being, in the sense that it indicates a self-subsistent entity. Therefore, a demonstration that aims to define an attribute by subsuming it under its middle term proves the attribute's being inasmuch as the demonstration defines it. Hence, the dual character of Aristotle's notion of demonstration may be resolved in light of the difference between his notion of being and the modern notion of existence. According to Aristotle, a demonstration establishes being rather than existence. Since being, unlike existence, is determined by a definition, Aristotle considers a procedure leading to definitions and explanations to be a proof that establishes being.

5.5 ARISTOTLE'S CONCEPT OF KNOWLEDGE

The foregoing analysis has led to the conclusion that the demonstrative procedure is aimed at defining the attribute of the conclusion by subsuming it under the appropriate middle term. In this interpretation, the demonstrative syllogism is a mediator between two types of definitions; it begins with a definition that specifies a perceptual understanding of the attribute and ends in a full-fledged demonstration that provides a conceptual understanding of this attribute. The demonstrative procedure having been interpreted in this way, Aristotle's concept of knowledge may be defined as a conceptualization. Aristotle distinguishes the state prior to the demonstration and full-fledged knowledge in terms of the known content. Through demonstrations a given content undergoes a qualitative modification, which turns it into a concept. However, from a modern point of view Aristotle's concept of knowledge does not appear to be knowledge at all. As I have already argued, Aristotle's modern commentators analyze "knowledge" in terms of the distinction between judgment and understanding. But this

distinction is inapplicable to Aristotle's concept of knowledge, since from that perspective, Aristotle's concept of knowledge would be construed as understanding rather than knowledge. To understand Aristotle's concept of knowledge, his theory of knowledge has to be seen from the perspective of the epistemology of his contemporaries.

Aristotle formed his theory of knowledge in answer to the skeptics of his time. In the *Posterior Analytics* I.2 he introduces his definition of knowledge as an alternative to the sophists' understanding of knowledge (71b9-10). Their skepticism is mainly based on the argument from the relativity of knowledge. Stating that man is the measure of all things, Protagoras conceives of the properties of an object as belonging to it only in relation to a human perspective. A thing is no more than what it appears to be from the perspective of human consciousness. In this view, nothing appears to be definite or self-identical; all qualities alter according to the reciprocal relation between an object and the knowing subject. In forming his theory of knowledge, Aristotle seeks to account for the absolute and essential properties of phenomena. Therefore, he traces the central objective of demonstration to definitions, which turn the diversified impressions into a fixed and determined concept.

The distinction made in the *Posterior Analytics* between knowledge of the fact and knowledge of the reason why can be construed as an attempt to avoid the relativity of cognitive contents. As I have shown in this chapter, these two modes of knowledge correspond respectively to Aristotle's distinction between things that are more familiar to us and things that are more familiar by nature. I have construed this distinction as a distinction between perceptual understanding, which is anchored in a human perspective, and conceptual understanding, which reveals real essences. By viewing demonstrations as mediating between perceptual understanding and conceptual understanding, Aristotle considers them a means of detaching the known content from the relative human perspective. By construing knowledge as a conceptualization, Aristotle is able to account for the absoluteness of knowledge.

Furthermore, Aristotle analyzes knowledge in terms of understanding; that is, knowledge differs from perception in its content. His identification of knowledge with understanding rather than with judgment reflects his attempt to detach knowledge from the human perspective. A judgment is a certain relation between a subject and a given content, so an analysis of knowledge in terms of judgment may give rise to a relativistic conception of knowledge, according to which the known content has its characteristics only in relation to the knowing subject. Since Aristotle considers knowledge a mode of understanding he dispenses with the judgmental aspect of knowledge. He

sees knowledge as a given content, while assimilating the relation of the knowing subject with the content. The absence of judgmental notions such as conviction and certainty enables Aristotle to counter the relativity of phenomena, and thereby to account for definite knowledge.

From a modern point of view Aristotle's theory of knowledge appears to be a theory of understanding and explanation. This interpretation stems from an attempt to construe Aristotle's theory of knowledge in light of the modern distinction between judgment and understanding. But if we abandon this distinction in favor of one between perception and thought, Aristotle's *Posterior Analytics* can be construed as a theory of knowledge, which forms the concept of knowledge in opposition to perception. Consequently, although Aristotle's *Posterior Analytics* appears, to the modern reader, to be a theory of explanation, it does not aim to provide such a theory. If we presuppose the distinction between perception and thought, rather than the distinction between judgment and understanding, knowledge and explanation become one and the same cognitive state. Knowledge is a cognitive state that extends beyond perception in its ability to provide a theoretical account of phenomena.

Conclusion

This study of Aristotle's theory of demonstration has been concerned with the theoretical relationship between Aristotle's logic and his theory of proof. In accounting for this relationship, I have attempted to uncover the presuppositions that relate Aristotle's notion of knowledge to the logical structure of demonstrations. This analysis concludes that Aristotle conceives of knowledge as conceptualization and the demonstrative procedure as establishing conceptual relations. Having established this contention through an examination of particular problems that *Posterior Analytics* arises, I turn now to consider it in more general terms.

Aristotle's theory of demonstration is based on the distinction between indemonstrable first principles and demonstrative knowledge. Apart from reflecting a logical distinction between primitive and derived propositions, this distinction refers to cognitive and ontological distinctions. First principles and derived propositions correspond to two different cognitive states, called by Aristotle "intellect" and "knowledge", respectively. Further, this distinction assumes an ontological distinction between a substance and an attribute; the former is known by means of the intellect and the latter is an object of demonstrative knowledge. The correspondence between the logical realm and the cognitive and ontological realms is expressed in the *Posterior Analytics* II.9:

Of some things there is something else that is their cause, of others there is not. Hence it is clear that in some cases what a thing is is immediate and a principle; and here one must suppose, or make apparent in some other way, both that they are and what they are (which the arithmetician does; for he supposes both what the unit is and that it is);¹ but in those cases which have a middle term and for which something else is the cause of their being, one can, as we have said, make them clear

¹ As I have shown in the first chapter the requirement to assume both "what it is" and "that it is" hold for the subject of the demonstrative conclusion. Hence the distinction, which is made here, between first principles and demonstrative knowledge is the distinction between a subject and an attribute.

through a demonstration, but not by demonstrating what they are. (93b21-28)

In this passage Aristotle draws the distinction between first principles and demonstrative knowledge from the distinction between subjects and attributes. The distinction between the modes of cognition appropriate for the attainment of first principles and demonstrative knowledge is explained by means the ontological distinction between entities whose cause is external and entities that include their cause within themselves. Demonstrative knowledge is possible with regard to attributes, since their dependent mode of existence entails their dependence on middle terms. Likewise, non-demonstrative knowledge is applicable to substances due to their ontological primacy, which manifests itself in the absence of external causes and hence of middle terms.

Despite this clear distinction, it seems that non-demonstrative knowledge, as well as demonstrative knowledge aim to expose similar contents; Aristotle considers the acquaintance with first principles, as well as demonstrative knowledge, to be knowledge of essences. This contention is stated explicitly in *Metaphysics* VII.17, where Aristotle concludes his discussion of substance. In that chapter Aristotle describes the inquiry into substances as analogous to the inquiry into attributes. According to this discussion, these two inquiries require different methods (1041b10) yet both reveal essences; the inquiry into substances reveals the essence of a substance (1041b8), such as a house, whereas the inquiry into attributes reveals the essence of a certain attribute, such as thunder (1041a28).

The correspondence between knowledge of substances and knowledge of attributes finds expression in Aristotle's distinction between induction and demonstration. My analysis of the notion of induction, employed in the *Posterior Analytics* II.19, has led to the conclusion that this notion is distinct from the argumentative notion of induction, employed in dialectical contexts. Following this contention, I have argued that the inductive process leading to the first principles results in universal concepts such as "man" or "animal". In analyzing the notion of induction discussed in the *Posterior Analytics* II.19, in light of Aristotle's discussions of induction in the *Prior Analytics* II.21 and the *Posterior Analytics* I.1, I have shown that this inductive process holds for ultimate particulars (i.e., the primary substances) yet it involves a generalization. As the primary substance is a compound of matter and form, the inductive process is interpreted here as leading to an apprehension of the form embodied in matter. In my interpretation induction is a mediator between two cognitive states: sensation and perception. It leads from the sensual acquaintance with the material manifold to an apprehension of the form, i.e., the essence. This process modifies the acquaintance with

particular entities by turning them into definite entities, which can be subsumed under a universal concept.

My interpretation of the demonstrative procedure ascribes an analogous role to demonstrations. When classifying the ingredients of a demonstrative branch of knowledge in the *Posterior Analytics* I.7 and 10, Aristotle views the attribute of the conclusion as the object that a demonstration proves and clarifies. In the second book of the *Posterior Analytics*, Aristotle analyzes the demonstrative procedure into two stages that correspond to his distinction between knowledge of the fact and knowledge of the reason why. In accounting for this distinction, I have shown that these two types of knowledge are tantamount to two types of definitions, which differ qualitatively. According to my interpretation, knowledge of the fact is attained by a verbal account that defines an attribute according to its perceptual characteristics, whereas knowledge of the reason why is attained by a definition that specifies conceptual characteristics. This latter type of definition is revealed indirectly through demonstration; for in demonstration the attribute of the conclusion is subsumed under the middle term, which specifies its essence. Hence, demonstration is a process that mediates between two types of understandings: perceptual and conceptual. Like induction, demonstration involves a modification of contents, which results in a conceptualization; induction leads from sensation to an apprehension of the essence, while demonstration leads from perceptual understanding to conceptual understanding.

Thus, Aristotle's general attitude to knowledge is determined by his attempt to apply the same notion of knowledge to both substances and attributes. As he considers knowledge of substances an acquaintance with the essence, attained by means of definitions, he conceives of demonstrative knowledge as an inquiry into the essence of a certain attribute. Such an inquiry is made through syllogistic inferences that explicate the content of the attribute. This approach determines Aristotle's view of the objects of knowledge in general, and the objects of mathematics in particular. Aristotle often characterizes the objects of mathematics as "the results of abstraction". This characterization reflects Aristotle's attempt to account for the possibility of mathematical knowledge, within the constraints of his ontology, according to which mathematical objects are attributes belonging to the category of quantity. The process of abstraction is interpreted here as a certain method of definition by which an attribute is regarded *as* having two of the characteristics of a substance – separability and essence. The formation of mathematical objects involves detaching the attribute from its subject and defining it according to genera and *differentiae*.

Aristotle's attempt to accommodate the objects of knowledge, i.e., attributes, to a notion of knowledge that is appropriate to substances finds

expression in his formalizations of mathematical proofs. Formulating these syllogistically, Aristotle attempts to deduce mathematical theorems from the definitions of the terms used in these theorems. For instance, in the *Posterior Analytics* II.11 he tries to deduce from the definition of a right angle the theorem that the angle in the semicircle is right. Generally, in formulating mathematical proofs syllogistically Aristotle presupposes that mathematical objects are results of abstractions. He considers the objects of proof universal concepts and attempts to deduce mathematical properties by explicating the content included in the definitions of these objects. By contrast, the mathematical objects that underlie Euclidean proofs are conceived of as the results of construction. As such, they differ from Aristotle's mathematical objects in two respects. First, they are not universal concepts but particular objects, to which construction operations can be applied. Second, the properties of these mathematical objects cannot be derived solely from their definitions but must also be derived from the application of certain constructions. These differences are manifestations of the fundamental difference between Aristotle and Euclid's conceptions of mathematical objects. As Aristotle considers mathematical objects quasi-substances, he sees them as having essential attributes that can be deduced from their essence. Euclid, by contrast, derives mathematical theorems from constructions, which exhibit the relation between a given mathematical object and a constructed mathematical object.

The discrepancy between Aristotle's theory of demonstration and mathematical practice reveals the normative character of Aristotle's theory of demonstration. The theory of demonstration presupposes the conception of knowledge that underlies Aristotle's notion of substance; it views knowledge as an acquaintance with an essence. This presupposition leads Aristotle to view the object of knowledge as a quasi-substance that has essential attributes. The application of the notion of substance to the objects of knowledge determines the form of reasoning that underlies the theory of demonstration. My analysis of Aristotle's theory of syllogism has shown that the notion of syllogistic entailment is narrower than the modern notion of logical entailment. Unlike the modern notion of entailment, syllogistic entailment is based on relations of subordination and coordination between syllogistic terms so it presupposes the classification system of genera and species. The theory of syllogism imposes on the theory of demonstration formal as well as conceptual constraints. The relations between species and genera are expressed in a predicative form, through which a conceptual relation between the subject and the predicate is stated.

While accounting for the logical structure of the first principles of demonstration, Aristotle accommodates them to the requirements of the

theory of syllogism. He holds that a demonstration requires three types of first principles: hypotheses, definitions, and axioms, of which only those of the first type serve as syllogistic premisses. In interpreting Aristotle's hypotheses I have argued that these propositions accord with the formal structure, as well as with the conceptual import of syllogistic premisses. Hypotheses are predicative propositions that imply being in the veridical sense; they can be true or false. This very feature of hypotheses implies that they should also express conceptual relations between subjects and predicates, as it follows from Aristotle's discussion of the principle of the excluded middle that being either true or false is characteristic for essential predications.

The other two types of first principles, definitions and axioms, guarantee the conceptual relations stated in hypotheses. The implicit assumptions of definitions and the law of the excluded middle are tantamount to the assumption that the subject of predication has an essential aspect. According to my analysis, definitions specify the meaning of a term, while the law of the excluded middle implicitly assumes that this meaning is definite. These assumptions, then, determine the essence of the subject term, and thereby enable predications to be essential. Hence, Aristotle's account of the first principles of demonstration is formed in accordance with the requirements imposed by the theory of syllogism.

Consequently, the theoretical foundation that links the various facets of Aristotle's theory of demonstration is to be found in his notion of substance. While developing the theory of substance in *Metaphysics* VII, Aristotle accounts for the ontological primacy of substances in cognitive terms; he characterizes the substance as an essence that can be defined. This ontological theory implies a notion of knowledge that identifies it with a definition that specifies an essence. Aristotle's theory of demonstration is motivated by the attempt to apply this notion of knowledge, which is appropriate to substances, to the objects of demonstrations, which are attributes.

The notion of knowledge that stems from Aristotle's conception of substance constitutes the basic difference between his theory of demonstration and modern axiomatization. Modern axiomatic systems are formed in accordance with epistemological assumptions about the priority of the judgment made by the knowing subject over the known content. These assumptions find expression in the modern notion of logical entailment, which is defined in terms of truth-values, and hence presupposes the primacy of judgment. By contrast, Aristotle's theory of demonstrative knowledge assigns little importance if any to the attitude of the knowing subject to the known content; knowledge is not analyzed in terms of judgment, justification, or certainty. Instead, Aristotle's endeavor in the *Posterior*

Analytics is to account for knowledge in terms of the known content; namely the relations subsisting between the subject and predicate of demonstrative conclusions. Unlike modern axiomatization, Aristotle's theory of demonstrative knowledge presupposes the primacy of understanding over judgment. This presupposition determines Aristotle's view of the form of reasoning that underlies his theory of demonstration. The theory of syllogism, unlike modern logical theories, analyzes the notion of logical entailment in terms of understanding rather than judgment. As it is a logic of understanding it accords with Aristotle's notion of knowledge.

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